But these are minor matters. This monograph is a welcome addition to the list of books to which one can send people who want to learn about modern real analysis.

REFERENCES


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Even when speaking to a group of differential geometers one cannot safely assume that everyone knows what an Einstein metric is. Why then would A. L. Besse write a 500 page book on the subject? With characteristic frankness he addresses that question in §B of the excellent nineteen page introduction (an impressive mathematical essay in its own right). This book is his response to the importunities received over his career to write a treatise on Riemannian geometry. This is the treatise, but with a focus and sense of purpose that make it far more exciting and fun than most weighty treatises ever are. Using Einstein spaces as his reference point, he can easily and naturally enter into some of the most exciting research activity of today: topology (the Poincaré conjecture), partial differential equations (Aubin-Yau solution of the Calabi problem, Yang-Mills theory), and the wonderful world of four dimensional geometry (self-duality and the Penrose construction), to name just three. In the final analysis, however, the author has a passion for Einstein metrics in their own right and his intention is to teach the reader what he knows about them, what great number of unanswered questions can be naturally asked about them, and why they merit enthusiastic study.

What then is an Einstein metric? A Riemannian metric $g$ on an $n$-dimensional manifold $M$ is a collection of positive definite inner products on the tangent spaces of $M$, one at each point, which vary smoothly in the sense that the inner product of any pair of $C^\infty$ vector fields on $M$ is a $C^\infty$ function on $M$. With respect to local coordinates $x^1, \ldots, x^n$ in $M$, $g = \sum g_{ij} dx^i dx^j$, where $g_{ij} = g_{ji}$ are $C^\infty$ functions. For example, in Euclidean space $\mathbb{R}^n$ where every tangent space is identified with $\mathbb{R}^n$ itself, the canonical Riemannian metric is $\sum (dx^i)^2$, the standard inner product on $\mathbb{R}^n$. In his inaugural address Riemann showed that his curvature tensor $R$ ($n^4$ local functions $R_{ijkl}$ with respect to local coordinates) provides the