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Commutator theory for congruence modular varieties, by Ralph Freese and Ralph McKenzie. London Mathematical Society Lecture Notes, vol. 125, Cambridge University Press, Cambridge, New York and Melbourne, 1987, 227 pp., $27.95. ISBN 0-521-34832-3

Ralph Freese and Ralph McKenzie are two of the outstanding North American contributors to the commutator; and after they have spent years revising and polishing preliminary manuscripts we finally have their fine book on the subject. The theme of the book is to show that the commutator is a versatile and useful tool; the authors succeed admirably in doing this. Roughly speaking, the commutator allows one to extend results for groups, rings and modules into the much larger domain of congruence modular varieties. Perhaps the best way to tell about the commutator is to sketch the background, and some of the results that have been achieved with it.

eUniversal algebra is mainly concerned with the study of algebraic structures (e.g., groups, lattices, rings, etc.), and with classes of such algebras defined by equations (e.g., groups of exponent 6, distributive lattices, idempotent rings, etc.). Some of the earliest results include the straightforward generalizations of the homomorphism theorems of group theory and ring theory. In the mid 1930s Birkhoff [2] made two fundamental contributions. First he showed that equational classes (classes of algebras defined by equations) were the same as varieties (classes of algebras closed under the formation of subalgebras, direct products and homomorphic images). Secondly he pointed out the importance of the lattice of congruences of an algebra. A congruence of an algebra **A** is an equivalence relation such that the obvious definition of a quotient algebra works; congruences can also be thought of as kernels of homomorphisms. The congruences of an algebra form a partially ordered set (under inclusion) which is a lattice.

In the 1960s and 1970s universal algebra received considerable stimulus from the tools and directions of logic. Jónsson [7] showed that one could use the ultraproduct construction from model theory to gain enormous insight into congruence distributive varieties, that is, varieties for which each of the algebras in the variety has a lattice of congruences satisfying the distributive law. The variety of lattices is probably the best known congruence