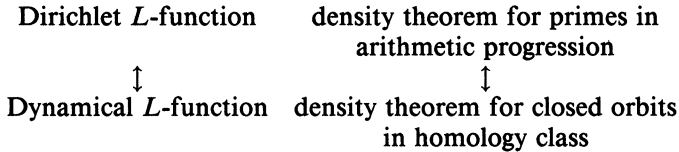


DYNAMICAL L -FUNCTIONS AND HOMOLOGY OF CLOSED ORBITS

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The phenomenon which we shall present in this note may be illustrated, in short, by the following diagram.



In the dynamical case, however, the “*ideal class group*” (= the first integral homology group) might have infinite order, so that some extra phenomena will be seen.

To fix our terminology, we let $\{\phi_t\}$ be a smooth, transitive Anosov flow [4] on a closed manifold X . We assume that ϕ_t has the weak-mixing property [17]. We denote by h the *topological entropy* of ϕ_t , and by μ a measure of *maximal entropy* on X , that is, an invariant probability measure whose metric entropy h_μ equals h . It is known that there exists exactly one measure with $h_\mu = h$ [20]. The *canonical winding cycle* Φ , which measures the average of “homological” direction in which the orbits of the flow are traveling, is defined by

$$\Phi(\omega) = \int_X \langle \omega, Z \rangle d\mu,$$

where ω is a closed 1-form, and Z is the vector field generating the flow. Since $\Phi(\text{exact forms}) = 0$, the linear map Φ yields actually a homology class in $H_1(X, \mathbf{R}) = \text{Hom}(H^1(X, \mathbf{R}), \mathbf{R})$ (see [16]).

We now classify closed orbits of the flow by means of the homology classes, and count the number of them. More generally, given a surjective homomorphism ψ of $H_1(X, \mathbf{Z})$ onto an abelian group H , we set for each $\alpha \in H$ and positive number x ,

$$\pi(x, \alpha) = \{\mathfrak{p}; \text{closed orbits with } \psi[\mathfrak{p}] = \alpha \text{ and } l(\mathfrak{p}) < x\},$$

$$\Pi(x, \alpha) = \#\pi(x, \alpha),$$

where $[\mathfrak{p}]$ denotes the homology class of \mathfrak{p} and $l(\mathfrak{p})$ the period. From now on, we shall identify the dual $H^\dagger = \text{Hom}(H, \mathbf{Z})$ with a subgroup in $H^1(X, \mathbf{Z})$ via the transpose of ψ . Set $b = \text{rank } H$.

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