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DYNAMICAL L-FUNCTIONS AND HOMOLOGY OF CLOSED ORBITS

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The phenomenon which we shall present in this note may be illustrated, in short, by the following diagram.

Dirichlet L-function	density theorem for primes in arithmetic progression
$\updownarrow$	$\updownarrow$
Dynamical L-function	density theorem for closed orbits in homology class

In the dynamical case, however, the "*ideal class group*" (= the first integral homology group) might have infinite order, so that some extra phenomena will be seen.

To fix our terminology, we let $\{\phi_t\}$ be a smooth, transitive Anosov flow **[4]** on a closed manifold X. We assume that ϕ_t has the weak-mixing property **[17]**. We denote by h the *topological entropy* of ϕ_t, and by μ a measure of *maximal entropy* on X, that is, an invariant probability measure whose metric entropy h_μ equals h. It is known that there exists exactly one measure with $h_\mu = h$ **[20]**. The *canonical winding cycle* Φ, which measures the average of "homological" direction in which the orbits of the flow are traveling, is defined by

$$\Phi(\omega) = \int_X \langle \omega, Z \rangle \, d\mu,$$

where ω is a closed 1-form, and Z is the vector field generating the flow. Since Φ(exact forms) $= 0$, the linear map Φ yields actually a homology class in $H_1(X, \mathbf{R}) = \mathrm{Hom}(H^1(X, \mathbf{R}), \mathbf{R})$ (see **[16]**).

We now classify closed orbits of the flow by means of the homology classes, and count the number of them. More generally, given a surjective homomorphism ψ of $H_1(X, \mathbf{Z})$ onto an abelian group H, we set for each $\alpha \in H$ and positive number x,

$\pi(x, \alpha) = \{\mathfrak{p}$; closed orbits with $\psi[\mathfrak{p}] = \alpha$ and $l(\mathfrak{p}) < x\}$,

$\Pi(x, \alpha) = \#\pi(x, \alpha)$,

where $[\mathfrak{p}]$ denotes the homology class of $\mathfrak{p}$ and $l(\mathfrak{p})$ the period. From now on, we shall identify the dual $H^\dagger = \mathrm{Hom}(H, \mathbf{Z})$ with a subgroup in $H^1(X, \mathbf{Z})$ via the transpose of ψ. Set $b = \mathrm{rank}\, H$.

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