RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

ON PROBLEMS OF U. SIMON CONCERNING MINIMAL SUBMANIFOLDS OF THE NEARLY KAEHLER 6-SPHERE

FRANKI DILLEN, LEOPOLD VERSTRAELEN AND LUC VRANCKEN

ABSTRACT. We classify the complete 3-dimensional totally real submanifolds with sectional curvature $K \geq \frac{1}{16}$ in the nearly Kaehler 6-sphere $S^6(1)$, and, as a corollary, we solve a problem for compact 3-dimensional totally real submanifolds of $S^6(1)$ related to U. Simon's conjecture for compact minimal surfaces in spheres.

1. The nearly Kaehler 6-sphere. It is well known that a 6-dimensional sphere S^6 does not admit any Kaehler structure, and whether S^6 does or does not admit a complex structure, as far as we know, is still an open question. However, using the Cayley algebra $\mathscr{C}$, a natural *almost complex structure* J can be defined on S^6 considered as a hypersurface in $\mathbf{R}^7$, which itself is viewed as the set $\mathscr{C}_+$ of the purely imaginary Cayley numbers (see, for instance, E. Calabi [**1**]). Together with the *standard metric* g on S^6, J determines a *nearly Kaehler structure* in the sense of A. Gray [**9**], i.e. one has $\forall X \in \mathscr{X}(S^6)\colon (\tilde{\nabla}_X J)(X) = 0$, where $\tilde{\nabla}$ is the Levi Civita connection of g. For reasons of normalization only, in the following we will always work with this nearly Kaehler structure on the sphere $S^6(1)$, of radius 1 and constant sectional curvature 1. The compact simple Lie group G_2 is the group of automorphisms of $\mathscr{C}$ and acts transitively on $S^6(1)$. Moreover, G_2 preserves both J and g.

2. Special submanifolds of $(S^6(1), g, J)$. With respect to J, two natural particular types of submanifolds M of $S^6(1)$ can be investigated: those which are *almost complex* (i.e. for which the tangent space of M at each point is invariant under the action of J) and those which are *totally real* (i.e. for

Received by the editors February 3, 1988.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 53C40, 53C15, 53C20.
The first and third authors received assistance from the Belgian National Science Foundation.