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ON THE GEOMETRY AND DYNAMICS OF DIFFEOMORPHISMS OF SURFACES

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PREFACE

This article was widely circulated as a preprint, about 12 years ago. At that time the *Bulletin* did not accept research announcements, and after a couple of attempts to publish it, I gave up, and the preprint did not find a home. I very soon saw that there were many ramifications of this theory, and I talked extensively about it in a number of places. One year I devoted my graduate course to this theory, and notes of Bill Floyd and Michael Handel from that course were circulated for a while. The participants in a seminar at Orsay in 1976–1977 went over this material, and wrote a volume [**FLP**] including some original material as well. Another good general reference, from a somewhat different point of view, is a set of notes of lectures by A. Casson, taken by S. Bleiler [**CasBlei**].

There are by now several alternative ways to develop the classification of diffeomorphisms of surfaces described here. At the time I originally discovered the classification of diffeomorphism of surfaces, I was unfamiliar with two bodies of mathematics which were quite relevant: first, Riemann surfaces, quasiconformal maps and Teichmüller's theory; and second, Nielsen's theory of the dynamical behavior of surface at infinity, and his near-understanding of geodesic laminations. After hearing about the classification of surface automorphisms from the point of view of the space of measured foliations, Lipman Bers [**Bers1**] developed a proof of the classification of surface automorphisms from the point of view of Teichmüller theory, generalizing Teichmüller's theorem by allowing the Riemann surface to vary as well as the map. Dennis Sullivan first told me of some neglected articles by Nielsen which might be relevant. This point of view has been discussed by R. Miller, J. Gilman, M. Handel and me.

The analogous theory, of measured laminations and 2-dimensional train tracks in three dimensions, has been considerable development. This has been applied to reinterpret some of Haken's work, to classify incompressible surfaces in particular classes of 3-manifolds in papers by me, Hatcher, Floyd, Oertel and others in various combinations. Shalen, Morgan, Culler and others have developed the related theory of groups acting on trees, and its relation to measured laminations, to define and analyze compactifications of representation spaces of groups in $SL(2, \mathbf{C})$ and $SO(n, 1)$; this has many interesting applications, including the theory of incompressible surfaces in 3-manifolds.

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