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A CRITIQUE OF NUMERICAL ANALYSIS

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1. Introduction. In this essay I want to raise the question "Is numerical analysis useful?". Most mathematicians, even those without any involvement in numerical computations, will think that the answer is obviously yes. It is common knowledge that computational methods are used daily by many members of the scientific community to solve problems that otherwise defy treatment. How can one seriously question the usefulness of something that has become a standard tool for many people?

To give substance to the question let me make a semantic distinction. I use the term *computational mathematics* to denote the wide spectrum of activities having to do with the approximate solution of scientific problems expressed through mathematical models. Typically, the equations arising from these models are differential or integral equations with no known closed form solution. For an approximate solution they must be discretized, that is, replaced by some finite system of equations that can be solved by algebraic methods. The whole process involves several phases and some quite distinct aspects. One is *numerical methodology* which considers ways of discretizing differential and integral operators and how best to solve the resulting finite systems. Another is *numerical analysis* which involves the rigorous study of the algorithms created by the methodology. The primary goal of analysis is to describe the relationship between the exact solution of the original equation and the approximate one obtained from its discretized version. It is numerical analysis in this narrower sense that I wish to examine here.

Even with this narrowed interpretation, the usefulness of numerical analysis is rarely questioned. Those who work in this area point out, with a great deal of justification, that analysis gives much insight into the nature of numerical methods and has contributed significantly to the widespread acceptance of numerical methodology. While some computational methods, such as relaxation and finite element techniques, were originated by engineers relying on physical insight, later analysis was crucial. Methods limited to special problems became general approaches as our theoretical understanding increased. In other instances the analysis suggested new methodologies. Numerical analysis has been instrumental in the design of effective numerical algorithms, and the effort expended has been repaid handsomely through the creation of a powerful tool for the solution of many important problems. Nevertheless, as I want to point out, this is not the end of the story. There are some fundamental issues that have been studied less thoroughly then they deserve, issues that grow in importance as scientists tackle more complex problems. There are open

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