1. Introduction. In [3], F. J. Dyson defined the rank of a partition as the largest part minus the number of parts. He let $N(m, t, n)$ denote the number of partitions of $n$ of rank congruent to $m$ modulo $t$, and he conjectured

$$N(m, 5, 5n + 4) = \frac{1}{5} p(5n + 4), \quad 0 \leq m \leq 4;$$

$$N(m, 7, 7n + 5) = \frac{1}{7} p(7n + 5), \quad 0 \leq m \leq 6,$$

where $p(n)$ is the total number of partitions of $n$ [1, Chapter 1]. These conjectures were subsequently proved by Atkin and Swinnerton-Dyer [2].

Dyson [3] went on to observe that the rank did not separate the partition of $11n + 6$ into 11 equal classes even though Ramanujan’s congruence

$$p(11n + 6) \equiv 0 \pmod{11}$$

holds. He was thus led to conjecture the existence of some other partition statistic (which he called the crank); this unknown statistic should provide a combinatorial interpretation of $\frac{1}{11} p(11n + 6)$ in the same way that (1.1) and (1.2) treat the primes 5 and 7.

In [4, 5], one of us was able to find a crank relative to vector partitions as follows:

For a partition $\pi$, let $\#(\pi)$ be the number of parts of $\pi$ and $\sigma(\pi)$ be the sum of the parts of $\pi$ (or the number $\pi$ is partitioning) with the convention $\#(\emptyset) = \sigma(\emptyset) = 0$ for the empty partition $\emptyset$ of 0. Let

$$V = \{(\pi_1, \pi_2, \pi_3) \mid \pi_1 \text{ is a partition into distinct parts,}$$

$$\pi_2, \pi_3 \text{ are unrestricted partitions}\}.$$  

We shall call the elements of $V$ vector partitions. For $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ in $V$ we define the sum of parts, $s$, a weight, $\omega$, and a crank, $r$, by

$$s(\vec{\pi}) = \sigma(\pi_1) + \sigma(\pi_2) + \sigma(\pi_3),$$

$$\omega(\vec{\pi}) = (-1)^{\#(\pi_1)},$$

$$r(\vec{\pi}) = \#(\pi_2) - \#(\pi_3).$$

We say $\vec{\pi}$ is a vector partition of $n$ if $s(\vec{\pi}) = n$. For example, if

$$\vec{\pi} = (5 + 3 + 2, 2 + 2 + 1, 2 + 1 + 1)$$

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