BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 17, Number 2, October 1987

0273-0979/87 $1.00 + $.25 per page

Asymptotics of high order differential equations, by R. B. Paris and A. D. Wood. Pitman Research Notes in Mathematics, vol. 124, Longman Scientific and Technical, Essex, and John Wiley, New York, 344 pp., $49.95. ISBN 0-470-20375-7

The theory of differential equations is an eminently applicable branch of mathematics. The differential equations that govern physical phenomena are mostly too complicated to allow a complete mathematical analysis. Usually, they involve partial derivatives of order higher than two and are nonlinear. The first part of the efforts at solving them then consists in drastic simplifications by assumptions that narrow down severely the scope of the results. Naturally, the simpler the new differential equations the more one can say about them. It is true that in the last few decades the development of fast computing machines has widened the range of problems that can be approached by numerical analysis, and, indirectly, the many unexpected phenomena discovered by numerical experiments have stimulated the interest in nonlinear equations.

There is, however, still much to be done in the long-studied theories of quite simple differential equations. Even for ordinary linear differential equations with analytic coefficients there exist more unanswered interesting questions—both from the theoretical point of view as well as for the applications—than most outsiders to the field realize. The simplest nontrivial such differential equations, the ones of order two, have been thoroughly explored, particularly in the nineteenth century. This literature fills many shelves, and even dry summaries of results with a few numerical tables added require several volumes.

The book under discussion goes beyond this classical material in that it concentrates on differential equations of order greater than two. It is not especially concerned with equations whose order is a *large* integer, although its title might suggest such a misunderstanding. In fact, the illustrative examples included—and I am glad that there are so many—are mostly of order not exceeding six. The differential equations are assumed to be ordinary, linear and homogeneous, and their coefficients are very special, simple analytic functions.

If these equations are written in the form

$$u^{(n)} - \sum_{r=0}^{p} c_r(z)u^{(r)} = 0, \tag{1}$$

where $0 \leqslant p \leqslant n$, the c_r are analytic functions of the complex variable z, and $u^{(r)} = d^r u/dz^r$, then a basic, very reassuring theorem guarantees that in a neighborhood of any point z_0 at which all c_r are holomorphic (i.e., regular