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Completely bounded maps and dilations, by Vern I. Paulsen. Pitman Research Notes in Mathematics, vol. 146, Longman Scientific and Technical, Essex, and John Wiley and Sons, New York, 1986, 187 pp., $38.95. ISBN 0-582-98896-9

This monograph uses C^*-algebraic techniques to study operator-theoretic problems. In particular, it uses the theory of completely positive and completely contractive maps to study dilations of operators. If T is a bounded linear operator on a Hilbert space H, then a dilation of T is a bounded linear operator S on a Hilbert space K containing H such that $Tx = PSx$ for each x in H, where P is the orthogonal projection of K onto H. Sometimes information about T can be obtained by dilating T to a "nice" operator S, using known facts about S, and then compressing back to H. Let $L(H)$ denote the algebra of all bounded linear operators on Hilbert space H. The two earliest dilation theorems are due to Naimark [3] and Sz.-Nagy [5]. Naimark proved that a regular, positive, $L(H)$-valued measure on a compact Hausdorff space can be dilated to be a spectral measure. Sz.-Nagy proved that if T belongs to $L(H)$ with the norm of T less than or equal to one, then T can be dilated to a unitary U such that $T^n x = PU^n x$ for all $n \geqslant 1$ and all x in H. Sz.-Nagy used this to prove von Neumann's inequality: If the norm of T is less than or equal to one and p is a polynomial, then $\|p(T)\| \leqslant \|p\|_\infty$, where $\|p\|_\infty$ denotes the uniform norm of p on the unit circle. Dilation theorems of various types are now standard in operator theory. See the book of Foiaş and Sz.-Nagy [6] or Halmos' Problem Book [2].

In 1955 W. F. Stinespring introduced a C^*-algebraic approach to dilation theory and used it to prove Naimark's theorem [4]. Besides the applications to