BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 17, Number 2, October 1987

THE TOPOLOGICAL STRUCTURE OF THE SPACE OF ALGEBRAIC VARIETIES

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The intention of this note is to announce some recent results concerning the homotopy type of the Chow variety of projective algebraic varieties. We begin by discussing the case of complex projective n-space $\mathbf{P}^n$.

For fixed integers $d \geq 1$ and p, $0 \leq p < n$, let $\mathcal{C}_{p,d}(\mathbf{P}^n)$ denote the set of effective p-cycles of degree d in $\mathbf{P}^n$. This is defined to be the family of all finite sums $c = \sum n_\alpha V_\alpha$, where for each α, n_α is a positive integer and $V_\alpha \subset \mathbf{P}^n$ is an irreducible subvariety of dimension p, and where $\text{degree}(c) = \sum n_\alpha \,\text{degree}(V_\alpha) = d$. The set $\mathcal{C}_{p,d}(\mathbf{P}^n)$ itself carries the structure of a projective algebraic variety. In particular it has a natural topology.

The spaces $\mathcal{C}_{p,d}(\mathbf{P}^n)$, for distinct values of d, are conventionally considered to be mutually disjoint. However, if we fix a "distinguished" p-dimensional linear subspace $l_0 \subset \mathbf{P}^n$, we obtain a natural sequence of topological embeddings,

$$\cdots \subset \mathcal{C}_{p,d}(\mathbf{P}^n) \subset \mathcal{C}_{p,d+1}(\mathbf{P}^n) \subset \cdots,$$

given at each level by mapping c to $c + l_0$. We can then consider the union of these spaces

$$\mathcal{C}_p(\mathbf{P}^n) = \lim_{d} \mathcal{C}_{p,d}(\mathbf{P}^n)$$

endowed with the weak limit topology. This is the topology in which a set C is closed if and only if the intersections $C \cap \mathcal{C}_{p,d}(\mathbf{P}^n)$ are closed for all d. Note that $\mathcal{C}_p(\mathbf{P}^n)$ is an abelian topological semigroup with unit $[l_0]$. Our first result is the following.

THEOREM 1. *For all n and p, $0 \leq p < n$, there is a homotopy equivalence*

$$\mathcal{C}_p(\mathbf{P}^n) \overset{\text{h.e.}}{\sim} K(\mathbf{Z}, 2) \times K(\mathbf{Z}, 4) \times \cdots \times K(\mathbf{Z}, 2(n-p)),$$

where $K(\mathbf{Z}, l)$ denotes the standard Eilenberg–Mac Lane space.

We recall that for an integer l and a finitely generated abelian group G, the Eilenberg–Mac Lane space $K(G,l)$ is a countable CW complex uniquely defined up to homotopy type by the requirement that: $\pi_l(K(G,l)) \cong G$ and $\pi_m(K(G,l)) = 0$ for $m \neq l$. The space $K(G,l)$ classifies the functor $H^l(\cdot\,; G)$, i.e., there is a natural isomorphism $H^l(X;G) \cong [X, K(G,l)]$ for any finite complex X. Under this equivalence the cohomology of $K(G,l)$ is carried over

Received by the editors January 23, 1987 and, in revised form, June 8, 1987.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 14C05; Secondary 55P15, 55Q52.
Research partially supported by NSF Grant No. DMS8602645 and the JSPS.

0273-0979/87 \$1.00 + \$.25 per page