

## THE STRUCTURE OF ALGEBRAIC THREEFOLDS: AN INTRODUCTION TO MORI'S PROGRAM

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**Introduction.** This article intends to present an elementary introduction to the emerging structure theory of higher-dimensional algebraic varieties. Introduction is probably not the right word; it is rather like a travel brochure describing the beauties of a long cruise, but neglecting to mention that the first half of the trip must be spent toiling in the stokehold. Perusal of brochures might give some compensation for lack of royal roads.

Having this limited aim in mind, the prerequisites were kept very low. As a general rule, geometry is emphasized over algebra. Thus, for instance, nothing is used from abstract algebra. This had to be compensated by using more results from topology and complex variables than is customary in introductory algebraic geometry texts. Still, aside from some harder results used in occasional examples, only basic notions and theorems are required.

Throughout the history of algebraic geometry the emphasis constantly shifted between the algebraic and the geometric sides. The first major step was a detailed study of algebraic curves by Riemann. He approached the subject from geometry and analysis, and gave a quite satisfactory structure theory. Subsequently the German school, headed by Max Noether, introduced algebra to the subject and problems arising from algebraic geometry substantially influenced the development of commutative algebra, especially the works of Emmy Noether and Krull.

During the same period the Italian school of Castelnuovo, Enriques, and Severi investigated the geometry of algebraic surfaces and achieved a satisfactory structure theory. Their work, however, lacked the Hilbertian rigor, and

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