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AMALGAMATIONS AND THE KERVAIRE PROBLEM

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ABSTRACT. Following S. Brick, a 2-complex X is called "Kervaire" if all systems of equations, with coefficients in arbitrary groups G and the attaching maps of X as the words in the variable letters, are solvable in an overgroup of G. An obstruction theory is developed for solving equations modeled on $Z = X \coprod_\Gamma Y$, where X and Y are Kervaire 2-complexes and Γ is a subgraph of $Z^{(1)}$, each connected component of which injects at the π_1-level into $\pi_1(Z)$. A 2-complex of the form $K\langle \vec{x}, \vec{y} |\ w(\vec{x}) = w'(\vec{y})\rangle$ is Kervaire, where $w(\vec{x})$ and $w'(\vec{y})$ are (not necessarily reduced) words which do not freely reduce to 1.

The Kervaire problem [**7**, p. 403] originally asked whether a nontrivial group can be killed by adjoining a single free generator and a single relator. This problem has been vastly generalized by Howie [**5**], who asked whether a system of equations over an arbitrary coefficient group G, whose words in the variable letters are the attaching maps of a 2-complex X with $H_2(X) = 0$, is solvable in an overgroup of G. It is convenient to introduce a terminology due to S. Brick [**1**] who calls a 2-complex X *Kervaire* iff all systems of equations over all coefficient groups G modeled on the attaching maps of X are solvable in an overgroup of G. Thus, e.g., the dunce hat $K\langle x|xx\bar{x}\rangle$ is Kervaire because Howie has shown that the equation $axbxc\bar{x} = 1$, with $a, b, c \in G$, can always be solved in an overgroup of G [**6**].

In this terminology, a nontrivial group can never be killed by adjoining a single free generator and a single relator iff the 2-complex $K\langle x|w(x)\rangle$ is Kervaire, where $w(x)$ is a word in x and x^{-1} whose exponent sum in x is ± 1.

For a 2-complex with one 2-cell $X = K\langle x_1, x_2, \ldots, x_n|w(\vec{x})\rangle$ Howie's problem can be shown (nontrivially) to imply that X is Kervaire iff $w(\vec{x})$ does not freely reduce to 1 (the "if" assertion is the nontrivial one here). Since $X = K\langle \vec{x}|w(\vec{x})\rangle$ can be easily shown to be Cockcroft iff $w(\vec{x})$ does not freely reduce to 1, Howie's problem for 2-complexes X with one 2-cell amounts to

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