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Complex manifolds and deformation of complex structures, by Kunihiko Kodaira, with an appendix by Daisuke Fujiwara. Translated by Kazuo Akao. Grundlehren der mathematischen Wissenschaften, vol. 283, Springer-Verlag, New York, Berlin, Heidelberg, and Tokyo, 1986, x + 465 pp., $48.00. ISBN 0-387-96188-7

In mathematics and science it is a familiar occurrence to have objects, such as systems of equations, depending on parameters. The investigation of this dependence goes under many names such as the study of bifurcations, or of unfoldings, or of deformations depending on the area. Historically and conceptually, the local deformation theory of compact complex manifolds has played a central role in the modern understanding of these phenomena. *Complex manifolds and deformation of complex structures* is a careful exposition of this local compact complex analytic deformation theory by one of its founders.

Deformation theory is as old as algebraic geometry. Riemann's theory of the theta divisor studied the fine structure of the family of algebraic line bundles on a given algebraic curve with special regard to the relation of the spaces of algebraic sections of the line bundles to the singularities of the theta divisors. (Of course, Riemann used the language of divisors and rational functions; see **[M3]**.) Out of this study of curves came the Riemann-Roch theorem and a count by Riemann of the number of parameters or moduli that an algebraic curve of genus g depends on. In the forties Teichmüller put the notion of moduli of Riemann surfaces on a rigorous basis, using ideas very special to the theory of one complex variable.

In two dimensions there was 19th-century work on families of curves on surfaces and some isolated work of Max Noether on the number of moduli certain algebraic surfaces should depend on. Further invariant theory was concerned with finding the global generic parameters or invariants that various natural families of algebraic geometric objects, such as hypersurfaces in $\mathbf{P}^n$, depend on (see **[Z** and **M2]**).

It is fair to say that at the start of the fifties no general results on deformations of complex or even algebraic manifolds of dimensions two or greater existed. Indeed, simple examples showed new phenomena that did not exist in one dimension, and made it unclear whether the set of such deformations has any structure, local or otherwise. It is worthwhile at this point to mention one of these. There are simple examples of maximal rank algebraic maps, $p\colon X \to \mathbf{P}^1$, from a three-dimensional projective manifold X down to the projective line $\mathbf{P}^1$ such that the fibre over the origin is a nontrivial $\mathbf{P}^1$ bundle over $\mathbf{P}^1$ and the fibre over every point other than the origin is isomorphic to $\mathbf{P}^1 \times \mathbf{P}^1$. This example of a really fine map exhibiting 'popping of the complex structure' at first sight does not seem to bode well for a local theory of continuous variation of complex structures.

In 1957 Frohlicher and Nijenhuis proved the very important result that if the first cohomology of the holomorphic tangent sheaf of a compact complex