

is  $C^\infty$  near  $\xi$  whenever  $P(f)$  is so. In some cases this can be seen because one can construct directly a pseudodifferential inverse; for this the large classes of pseudodifferential operators described in Chapter 19 are a very sharp tool. In other cases one cannot construct directly a pseudodifferential inverse, but one can prove microhypoellipticity by arguments using both microlocal geometry and a priori estimates. Such is the case for operators “of Kolmogorov type”:  $X_0 + \sum X_i^2$ , where the  $X_j$  are vector fields whose Lie algebra spans the whole space at every point.

Chapter 24 contains the theory of the mixed Cauchy-Dirichlet problem for second-order differential operators (i.e., the study of the evolution in time of the solution of the wave equation in a bounded domain, with some reflection condition on the boundary). The existence of solutions has been proved by techniques using energy inequalities. The precise study of the singularities of the solutions and of their propagation requires the full arsenal of microlocal analysis.

The last chapter (30) is on scattering theory for long-range potentials (short-range scattering is dealt with in Volume II). The typical example is the theory of  $H = \sum \partial^2 / \partial x_j^2 + V(x)$ , where the potential  $V$  does not decay fast enough at infinity (e.g.  $V = O(1/|x|)$ ). The aim is to intertwine the part of  $H$  with continuous spectrum with the Laplace operator  $\sum \partial^2 / \partial x_j^2$  (i.e., prove that nonbounded particles behave at infinity as free particles). One of the key ingredients of the theory is the construction of a distorted Fourier transformation adapted to  $H$ , i.e., of a family of approximate solutions of  $H(f_\xi) = -|\xi|^2 f_\xi$  which behave at infinity asymptotically as  $\exp(-ix \cdot \xi)$ . The same ideas of microlocal analysis are used, now applied to asymptotic expansions when  $x \rightarrow \infty$ .

The lines above only give a very short idea of the contents of the book. I at least hope they will be motivation to read it. Each chapter of the book also contains an introduction, which describes with more details the contents and methods of the chapter, and a bibliographical and historical notice. The book also contains a very complete bibliography. It is a superb book, which must be present in every mathematical library, and an indispensable tool for all—young and old—interested in the theory of partial differential operators.

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*Minimal surfaces and functions of bounded variation*, by Enrico Giusti, *Monographs in Mathematics*, Vol. 80, Birkhäuser, Boston-Basel-Stuttgart, 1984, xii + 240 pp., \$39.95. ISBN 0-8176-3153-4

Among all surfaces spanning a given boundary is there one of least area? Such problems have sometimes been called collectively *the problem of Plateau* in honor of a nineteenth-century physicist who wrote a treatise on equilibrium