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Representations of compact Lie groups, by Theodor Bröcker and Tammo tom Dieck, Graduate Texts in Mathematics, Vol. 98, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985, x + 313 pp., $39.00. ISBN 0-387-13678-9

Compact groups are marvelous. In many branches of mathematics and physics they arise quite by nature, frequently, but not always in the form of Lie groups. They have a long history—long anyhow by the standards of functional analysis. Yet they still provide stimulation for current research. The field of compact groups offers an excellent testing ground for analysts, topologists, and algebraists alike as soon as they care to verify their results in the case of compact groups.

There are good theoretical reasons for this. For example, the entire topological structure of a connected locally compact group is carried (up to a topological copy of $\mathbf{R}^n$ as direct factor) by a compact subgroup which is unique up to conjugation. In the representation theory of locally compact groups, compact groups are not only a model after which the more delicate results of the general theory are fashioned, but they are frequently a determining element in such processes as the construction of induced representations. The best part of the duality theory of locally compact abelian groups concerns the bijective correspondence of compact abelian and discrete abelian groups, and this situation remains quite significant even in the simplest situations involving compact abelian Lie groups on the one hand and finitely generated abelian groups on the other.

The theory of compact groups pivots around the subclass of compact Lie groups. (Dieudonné in 1973: "*Les groupes de Lie sont devenus le centre des mathématiques. On ne peut rien faire de sérieux sans eux.*" [**7**]) In this area we find the richest treasures of the whole theory. Compact groups yielded the first partial solution to Hilbert's Fifth Problem which stipulated that any locally euclidean topological group should be a Lie group [**17**]. Soon it emerged that every compact group can be approximated in a controlled fashion by compact Lie groups, a fact which was distilled into its modern form by the mid-thirties [**19**]. It is true that the basic representation theory of compact groups rests on real analysis and functional analysis; but the representation theory of compact *Lie* groups is at the root of the representation theory of arbitrary compact groups in its more subtle aspects. Compact Lie groups and their homogeneous spaces are prime examples of compact smooth manifolds. It is here that the fundamental concepts of the calculus of several variables and of differential geometry may be tested and brought to good use. Many of the elementary aspects of differential geometry are based on properties of the orthogonal groups. The vector product in $\mathbf{R}^3$ which pops up in so many elementary mathematics and physics courses remains mysterious until it is recognized as