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Prescribing the curvature of a Riemannian manifold, by Jerry L. Kazdan, CBMS Regional Conference Series in Mathematics, vol. 57, American Mathematical Society, Providence, R. I., 1985, vii + 55 pp., $12.00. ISBN 0-8218-0707-2

It is generally believed that a most interesting area in nonlinear partial differential equations lies in the study of special equations, particularly those arising from geometry and physics. The monograph under review is an account of problems on the existence of a Riemannian metric with given curvature conditions. It contains some of the most important results in mathematics in recent years.

There are all kinds of curvatures: Gaussian curvature, scalar curvature, Ricci curvature, Riemannian sectional curvature, etc. When any one of them is prescribed, we get a system of partial differential equations on the fundamental tensor of the Riemannian metric. The problems have a meaning for a manifold without boundary, giving rise to some problems attractive because of their simplicity. But Chapter IV of these notes gives a treatment of some recent developments on boundary-value problems.

Even for the Gaussian curvature there are unanswered questions. To be the Gaussian curvature of a compact surface M^2, a function $K \in C^\infty(M^2)$ must satisfy a sign condition forced by the Gauss-Bonnet theorem. Kazdan and Warner proved that this is sufficient. It would be interesting to prove this by a conformal transformation of a given metric g_0 on M^2. There will be no difficulty if the Gaussian curvature K_0 of g_0 is negative. For $K_0 > 0$ there are further necessary conditions and it is not known whether they are sufficient. Even for the two-sphere $M^2 = S^2$ it is not known whether one could obtain a metric of constant curvature through the conformal transformation of a given one.

The simplest generalization of the Gaussian curvature to higher dimensions is the scalar curvature, which is a scalar invariant. By applying the Bochner