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ON DISCRETE CHAMBER-TRANSITIVE AUTOMORPHISM GROUPS OF AFFINE BUILDINGS

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1. Introduction. Let Δ be the affine building of a simple adjoint algebraic group $\mathcal{G}$ of relative rank ≥ 2 over a locally compact local field K. Let $\operatorname{Aut}\Delta$ (resp. $\Sigma\operatorname{Aut}\Delta$) denote the group of type-preserving (resp. of all) automorphisms of Δ. Note that $\Sigma\operatorname{Aut}\Delta$ contains the group $\mathcal{G}(K)$ of K-rational points of $\mathcal{G}$. We will be interested in discrete subgroups of $\operatorname{Aut}\Delta$ which are chamber-transitive on Δ. It is extremely rare that such groups exist and, as can therefore be expected, exceptions are interesting phenomena; our purpose is to list them all (see the theorem below). In order to describe them we must first introduce some notation.

Let f be a quadratic form in n variables over $\mathbf{Q}_p$ with coefficients in $\mathbf{Z}$. We let $\mathrm{P}\Omega(f, \mathbf{Z}[1/p])$ denote the intersection $\mathrm{PSO}(f, \mathbf{Q}_p)' \cap \mathrm{PGL}(n, \mathbf{Z}[1/p])$ within $\mathrm{PGL}(n, \mathbf{Q}_p)$, and similarly $\mathrm{PGO}(f, \mathbf{Z}[1/p]) = \mathrm{PGO}(f, \mathbf{Q}_p) \cap \mathrm{PGL}(n, \mathbf{Z}[1/p])$. In the following list, Γ will always be a chamber-transitive subgroup of $\operatorname{Aut}\Delta$. The fundamental quadratic form (over $\mathbf{Z}$) of the root lattice of type A_n, B_n, E_n, normalized so that the long roots have squared length 2, will be denoted by a_n, b_n, e_n, respectively; note that b_n is $\sum_1^n x_i^2$.

(i) Let $f = e_8, b_7, a_6, b_6, e_6$, or a_5, and let Δ be the affine building of $\mathrm{PSO}(f, \mathbf{Q}_2)$. Here Γ can be any group between $\Gamma_{\min} = \mathrm{P}\Omega(f, \mathbf{Z}[1/2])$ and $\Gamma_{\max} = \mathrm{PGO}(f, \mathbf{Z}[1/2]) \cap \operatorname{Aut}\Delta$. The quotient $\Gamma_{\max}/\Gamma_{\min}$ is elementary abelian of order 1, 1, 1, 4, 2, or 2, respectively, and $\Gamma_{\max}$ is generated by $\Gamma_{\min}$ and reflections.

(ii) Let $f = b_5, e_6$, or $b_6' = \sum_1^5 x_i^2 + 3x_6^2$, and let Δ be the building of $\mathrm{PSO}(f, \mathbf{Q}_3)$. The group $\Gamma_{\max}(f) = \mathrm{PGO}(f, \mathbf{Z}[1/3]) \cap \operatorname{Aut}\Delta$ has 3, 5, or 9 conjugacy classes of chamber-transitive subgroups Γ. Passage mod 2 maps $\Gamma_{\max}(b_5)$ onto the symmetric group S_5, and the preimages in $\Gamma_{\max}(b_5)$ of S_5, A_5, or a group of order 20 form the 3 desired conjugacy classes of groups Γ. The forms e_6 and b_6' are rationally equivalent, and hence the buildings they define over $\mathbf{Q}_3$ are the "same"; with suitable identifications of buildings and groups, $\Gamma^\flat = \Gamma_{\max}(e_6) \cap \Gamma_{\max}(b_6')$ has index 27 in $\Gamma_{\max}(e_6)$ and index 2 in $\Gamma_{\max}(b_6')$. Passage mod 2 maps $\Gamma_{\max}(e_6)$ onto $\mathrm{PGO}(5,3)$, and the preimages in $\Gamma_{\max}(e_6)$ of the 5 different classes of flag-transitive subgroups of $\mathrm{PGO}(5,3)$ (cf. [S]) form the 5 desired conjugacy classes of groups Γ, exactly 3 of which have members in $\Gamma^\flat$. The 6 remaining conjugacy classes of chamber-transitive subgroups of $\Gamma_{\max}(b_6')$ not having members in $\Gamma_{\max}(e_6)$ consist of groups having index 1 or 2 in $\langle \Gamma, r\rangle$ for one of the chamber-transitive subgroups Γ of $\Gamma^\flat$, where r is the reflection $x_6 \mapsto -x_6$.

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