BOOK REVIEWS

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Approximation of Hilbert space operators, Volume I, by Domingo Herrero, Pitman Publishing Inc., Boston, 1982, xiii + 255 pp., $23.95. ISBN 0-273-08579-4

Approximation of Hilbert space operators, Volume II, by Constantin Apostol, Lawrence Fialkow, Domingo Herrero and Dan Voiculescu, Pitman Publishing Inc., Boston, 1984, x + 524 pp., $29.95. ISBN 0-273-08641-3

1. Introduction. The theme of the books under review is *approximation*; that is, how well do simple models of operators approximate larger, less understood classes? This usually means approximation in the operator norm, but it may well ask for more. For example, one might ask that the error of estimation be compact. As the set of compact operators is the only proper, closed, two-sided ideal in the algebra $\mathcal{B}(\mathcal{H})$ of bounded operators on a separable Hilbert space $\mathcal{H}$, this is a natural constraint.

The study of bounded operators on Hilbert space has often been motivated by linear algebra. There is a popular, but naive, notion that finite dimensions are well understood. Operator theorists are keen to find the infinite-dimensional analogues of the finite-dimensional results. But they obtain even more pleasure when they find out why such analogues cannot hold.

Halmos, in his role as the master popularizer of this subject, has asked many questions about approximation of operators. In particular, his famous *Ten problems in Hilbert space* **[Ha2]** has provoked some of the most important work in this area. (See **[Ha3]** for the progress report.) Here we will mention two examples for motivation.

Everyone knows that a normal matrix can be diagonalized. This is not the case in infinite dimensions. For example, on $L^2(0,1)$, the operator given by $(Mf)(x) = xf(x)$ has no eigenvalues. Halmos asked: Is every normal operator the sum of a diagonal operator and a compact one? The answer (see §2) has had many ramifications.

On a finite-dimensional space, the set of nilpotent matrices is closed, and consists of all matrices with spectrum $\{0\}$. In infinite dimensions, the operators with spectrum $\{0\}$ (called *quasinilpotents*) are the operators satisfying

$$\lim_{n\to\infty} \|T^n\|^{1/n} = 0.$$