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PERIODIC GEODESICS OF GENERIC NONCONVEX DOMAINS IN $\mathbf{R}^2$ AND THE POISSON RELATION

VESSELIN M. PETKOV AND LUCHEZAR STOJANOV

1. Introduction. Let $\Omega \subset \mathbf{R}^n$, $n \geq 2$, be a bounded connected domain with C^∞ smooth boundary $\partial\Omega$. Consider the eigenvalues $\{\lambda_j^2\}_{j=1}^\infty$ corresponding to the Dirichlet problem for the Laplacian

$$(1) \qquad -\Delta u = \lambda^2 u \text{ in } \Omega, \qquad u = 0 \text{ on } \partial\Omega.$$

The Poisson relation for $\sigma(t) = \sum_j \cos\lambda_j t$ has the form

$$(2) \qquad \operatorname{sing\,supp} \sigma(t) \subset \bigcup_{\gamma \in \mathcal{L}_\Omega} \{-T_\gamma\} \cup \{0\} \cup \bigcup_{\gamma \in \mathcal{L}_\Omega} \{T_\gamma\}.$$

Here $\mathcal{L}_\Omega$ is the union of all generalized periodic geodesics γ in $\overline{\Omega}$, including those lying entirely on $\partial\Omega$, and T_γ is the period (length) of γ (see [**1**]). Generalized geodesics are projections on $\overline{\Omega}$ of the generalized bicharacteristics of $\partial_t^2 - \Delta$, introduced by Melrose and Sjöstrand [**6**]. We have proved in [**8, 9**] that for generic strictly convex domains in $\mathbf{R}^2$ the relation (2) becomes an equality and the spectrum of (1) determines the lengths of all periodic geodesics (see [**5**] for related results). The purpose of this announcement is to prove the same result for generic nonconvex domains in $\mathbf{R}^2$.

2. Main results. In the analysis of (2) for nonconvex domains three difficulties appear: (A) the existence of periodic geodesics having gliding segments on $\partial\Omega$ and linear segments in the interior of Ω, (B) some linear segment l of a periodic geodesic could be tangent to $\partial\Omega$ at some interior point of l, (C) the linear Poincaré map P_γ of a reflecting periodic geodesic γ could contain in its spectrum 1 or $\sqrt[p]{1}$ with $p \in \mathbf{N}$. We refer to [**3**] for the precise definition of reflecting geodesics and the related Poincaré map. A linear segment is a set $l = [x, y] = \{z; z = \alpha x + (1-\alpha)y,\ 0 \leq \alpha \leq 1\}$, while a gliding segment is an arc $\delta \subset \partial\Omega$. We show below that generically for domains in $\mathbf{R}^2$ the phenomena (A), (B), (C) cannot occur. We begin by assuming $\Omega \subset \mathbf{R}^2$.

Set $\partial\Omega = X$ and consider the space $C^\infty_{\text{emb}}(X, \mathbf{R}^2)$ of all C^∞ smooth embeddings of X into $\mathbf{R}^2$ with the Whitney topology [**2**]. For $f \in C^\infty_{\text{emb}}(X, \mathbf{R}^2)$ we denote by $\Omega_f \subset \mathbf{R}^2$ the bounded domain with boundary $f(X)$. A set $\mathcal{R} \subset C^\infty_{\text{emb}}(X, \mathbf{R}^2)$ will be called residual if $\mathcal{R}$ is a countable intersection of open dense sets.

THEOREM 1. *Let Ω be a domain with boundary X. There exists a residual set $\mathcal{R} \subset C^\infty_{\text{emb}}(X, \mathbf{R}^2)$ such that for every $f \in \mathcal{R}$ there are no generalized periodic geodesics $\gamma \in \mathcal{L}_{\Omega_f}$ having at least one gliding segment on $f(X)$ and*

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