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ON THE ZEROS OF MAASS WAVE FORM L-FUNCTIONS

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In [**1**], Epstein, Sarnak, and I gave some general results concerning the zeros of an L-function attached to an even Maass wave form for $\Gamma = \mathrm{PSL}(2, \mathbf{Z})$. The main result was that such an L-function has many zeros on its critical line. In fact, we showed that the analogue of the Hardy-Littlewood theorem for the Riemann zeta function holds for these L-functions as well. In this note I announce the next step: the analogue of Selberg's theorem. That is, a positive proportion of the zeros lie on the critical line. This then significantly extends the class of Dirichlet series for which this is known. To date, this included only the classical Dirichlet L-series [**8**] including the Riemann zeta function [**7**], and the L-functions attached to holomorphic cusp forms of Γ [**2, 4, 5**]. (The proofs in the last case and the present case probably extend to cusp forms on congruence subgroups as well, but this has not been thoroughly verified.) Effectively then, this result applies to all cusp forms for GL(2).

Let us formulate the theorem more explicitly. We begin with Γ acting on the upper half-plane $\mathcal{H} = \{z \in \mathbf{C}: \operatorname{Im} z > 0\}$ via linear fractional transformations. A Maass wave form is a Γ-automorphic function on $\mathcal{H}$ which is in $L^2(\Gamma\backslash\mathcal{H})$ and is simultaneously an eigenfunction of the Laplacian and all the Hecke operators. That is, f satisfies

$$\text{(i)} \qquad \int_{\Gamma\backslash\mathcal{H}} |f(z)|^2 \frac{dxdy}{y^2} < \infty,$$

$$\text{(ii)} \qquad \Delta f = (\frac{1}{4} + r^2)f, \quad \Delta = -y^2(\partial_x^2 + \partial_y^2),$$

$$\text{(iii)} \qquad f(\gamma z) = f(z), \quad \gamma \in \Gamma, \quad z \in \mathcal{H},$$

$$\text{(iv)} \qquad T_n f = a(n) f, \quad n \geq 1.$$

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