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*On a new method of analysis and its applications*, by Paul Turán, John Wiley & Sons, Inc., Somerset, New Jersey, 1984, xvi + 584 pp., $49.95. ISBN 0471-89255-6

This book has a remarkable history. The present edition was announced twenty-five years before its publication. The first version of Turán's book was published in 1953 in Hungarian and in German. An improved Chinese edition followed in 1956. The theory was developed so rapidly that only a few years later these editions were out of date. In 1959, in a footnote of an article [**8**], Turán announced a completely rewritten English edition. He wanted to cover all the theory and major applications of the power sum method which bears his name. A series of new inequalities and applications, found by Turán and other mathematicians, among whom Atkinson, de Bruijn, Cassels, Erdős, Gaier, Halász, Pintz, van der Poorten, Sós, Stark, and Uchiyama, led him to extend the manuscript and to rewrite old parts of the manuscript repeatedly. At his death on September 26, 1976 he left a carefully organized manuscript comprising 57 sections. Sections 1–37 (except for Section 26) were in final form. For the remaining sections he had indicated the intended contents. These sections were written by Halász and Pintz following Turán's intentions as far as possible. Mrs. Sós took care of the further coordination. These efforts resulted in the publication of the English edition, eight years after Turán's death. The present book has 584 pages compared to 196 pages of the German edition. It is a unified treatise of the theory to which Turán devoted a considerable part of his life.

The roots of Turán's method are the roots of prime number theory. The Prime Number Theorem states that the number $\pi(x)$ of primes at most $x$ is asymptotically equal to

$$\operatorname{li} x := \int_2^x \frac{dt}{\log t} \sim \frac{x}{\log x}.$$

The theorem is equivalent to the nonvanishing of the Riemann zeta function $\zeta(s) = \zeta(\sigma + it)$ at the line $\sigma = 1$. The still unproved Riemann Hypothesis states that $\zeta(s) \neq 0$ for $\sigma > 1/2$. For $\sigma > 1$ we have $\zeta(s) = \sum_{j=1}^{\infty} j^{-s}$ and $\zeta(s) \neq 0$ because of Euler's product formula for $\zeta(s)$. In 1911 H. Bohr [**2**] proved the surprising fact that

$$\inf_{\sigma > 1} |\zeta(\sigma + it)| = 0.$$

In the same year Bohr [**1**] solved the problem of Lindelöf whether or not $\zeta(s)$ is bounded for $\sigma > 1$, $|t| \geqslant 1$ by proving its unboundedness. Bohr's proofs are similar, but for the latter result Bohr used Dirichlet's theorem and for the former Kronecker's theorem from the theory of diophantine approximation. An essential difference between these theorems is that there is a localization in Dirichlet's, but not in Kronecker's theorem. Such a localization enables one to