BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 2, April 1986

REALIZING SYMMETRIES OF A SUBSHIFT OF FINITE TYPE BY HOMEOMORPHISMS OF SPHERES

BY J. B. WAGONER[1]

Let A be a finite, irreducible, zero-one matrix and let $\sigma_A: X_A \to X_A$ be the corresponding subshift of finite type [**F**]. Recall from [**F**] that a Smale diffeomorphism is one with a hyperbolic zero-dimensional chain recurrent set. A well-known theorem of Williams-Smale [**Wi**] says that there is a Smale diffeomorphism $F_A: S^3 \to S^3$ so that σ_A is topologically conjugate to the restriction of F_A to the basic set of index one occurring as part of the spectral decomposition. Let $\mathrm{Aut}(\sigma_A)$ denote the group of symmetries of σ_A—that is, the group of homeomorphisms of X_A which commute with σ_A. Here is the corresponding global realization result for these symmetries.

THEOREM. *Assume $4 < q$ and let $1 < e < q - 2$. Then there is a Smale diffeomorphism $F_A: S^q \to S^q$ with a basic set Ω_e of index e (along with other basic sets of index 0, $e+1$, q) together with a topological conjugacy between σ_A and $F_A|\Omega_e$ so that, given any symmetry g in $\mathrm{Aut}(\sigma_A)$, there is a homeomorphism $G: S^q \to S^q$ satisfying*
(A) *G commutes with F_A on all of S^q,*
(B) *$G|\Omega_e = g$ under the identification between $\mathrm{Aut}(F_A|\Omega_e)$ and $\mathrm{Aut}(\sigma_A)$.*

The motivation and the idea for the proof of this geometric result came by analogy from algebraic K-theory and pseudo-isotopy theory. The proof uses Williams' notion of strong shift equivalence [**W1, F**], the contractible simplicial complex P_A of topological Markov partitions for σ_A [**W1**], and structural stability for Smale diffeomorphisms [**R, Ro**]. We would like to thank C. Pugh for useful discussions about the stability theorem.

The group $\mathrm{Aut}(\sigma_A)$ is often rather large. For example, $\mathrm{Aut}(\sigma_2)$ for the Bernoulli 2-shift σ_2 has been known [**H**] for some time to contain every finite group and to have elements of infinite order not a power of σ_2. Recently, Boyle and Lind have shown it contains the free nonabelian group on infinitely many generators. Therefore, the group of *homeomorphisms* of S^q commuting with a certain F_2 is large when $4 < q$. Incidentally, at the present time not much is really known about the structure and other algebraic or homological properties of $\mathrm{Aut}(\sigma_2)$. For some information see [**BK**] or [**W1**]. An open and long-standing conjecture is that $\mathrm{Aut}(\sigma_2)$ is generated by σ_2 and elements of finite order.

Here is a rough idea of the proof of the Theorem. The details will appear in [**W2**]. Let P be an $m \times m$ zero-one matrix and let Q be an $n \times n$ zero-one matrix. Suppose there is an $m \times n$ zero-one matrix R and an $n \times m$ zero-one

Received by the editors September 30, 1985.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 34C35, 20B27.
[1]Partially supported by the NSF.