BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 2, April 1986

INTERSECTIONS OF HIGHER-WEIGHT CYCLES OVER QUATERNIONIC MODULAR SURFACES AND MODULAR FORMS OF NEBENTYPUS

BY B. BRENT GORDON[1]

In 1976 Hirzebruch and Zagier [**6**] computed the pairwise intersection multiplicities for a family of algebraic cycles $(T_n^c)_{n\in\mathbf{N}}$ in the Hilbert modular surface associated to $\mathbf{Q}(\sqrt{p})$, where p is a prime congruent to 1 mod 4, and showed that the generating function for those intersection multiplicities $\sum_{n=0}^{\infty}(T_m^c \cdot T_n^c)e[n\tau]$ was an elliptic modular form of weight 2 and Nebentypus for $\Gamma_0(p)$. Shortly afterwards Zagier [**22**] observed that if certain weighting factors were attached to the intersection numbers, then the new generating function was again an elliptic modular form of the same level and Nebentypus but now of higher weight. Thus he was led to ask if these weighted intersection numbers could also be realized as the ordinary geometric intersection multiplicities of some algebraic cycles in some appropriate homology theory for the Hilbert modular surface. The purpose of this announcement is to describe an answer to Zagier's question for quaternionic modular surfaces. By combining some ideas of Millson [**14**] about higher-weight cycles in torus bundles over locally symmetric spaces with the notion of an algebraically defined subspace in the cohomology of a Kuga fiber variety (cf. [**4**]), we are able to associate to each Hirzebruch-Zagier cycle T_n in a quaternionic modular surface $\mathcal{X}$ an algebraic cycle $\mathcal{T}_n$ in a family of abelian varieties $\mathcal{A}$ over $\mathcal{X}$ such that: (a) The pairwise intersection multiplicities $(\mathcal{T}_m \cdot \mathcal{T}_n)$ of these cycles have precisely the same form as Zagier's weighted intersection numbers (Theorem 1), and (b) the generating function for intersection numbers $\sum_{n=0}^{\infty}(\mathcal{T}_m \cdot \mathcal{T}_n)e[n\tau]$ is an elliptic modular form of higher weight and Nebentypus for an appropriate $\Gamma_0(N) \subset \mathrm{SL}_2(\mathbf{Z})$ (Theorem 2).

Tong [**20**] also looked at Zagier's question. In 1979 his response was to associate to each Hirzebruch-Zagier cycle a current in the cohomology of the Hilbert modular surface with coefficients in a complex vector bundle and show, using the theory of [**19**], that the intersection multiplicities of these currents coincided with Zagier's weighted intersection numbers. Now it is no coincidence that the cohomology classes represented by the cycles $\mathcal{T}_n$ live in a subspace $H^\bullet(\mathcal{M})$ of $H^\bullet(\mathcal{A}, \mathbf{Q})$ which is isomorphic to the vector-bundle-valued cohomology that Tong worked with (cf. Lemma 1 below), and that under such an isomorphism the cycles $\mathcal{T}_n$ correspond to his currents. The point is that the whole picture can be realized algebraically inside of the variety $\mathcal{A}$ (Proposition 1), so that one could start a priori with the "motive" $H^\bullet(\mathcal{M})$. Moreover, the

Received by the editors August 25, 1985.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 10D21, 10D12.
[1] Research partially supported by the SFB "Theoretische Mathematik" and the Max-Planck-Institut für Mathematik, Bonn, and the University of Maryland.

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