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COMPACT RIEMANNIAN MANIFOLDS WITH POSITIVE CURVATURE OPERATORS

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The Riemann-Christoffel curvature tensor R of a Riemannian manifold M determines a *curvature operator*

$$\mathcal{R}: \Lambda^2 T_p M \to \Lambda^2 T_p M,$$

where $\Lambda^2 T_p M$ is the second exterior power of the tangent space $T_p M$ to M at p, by the explicit formula

$$\langle \mathcal{R}(x \wedge y), z \wedge w \rangle = \langle R(x,y)w, z \rangle.$$

M is said to have *positive curvature operators* if the eigenvalues of $\mathcal{R}$ are positive at each point $p \in M$. Meyer used the theory of harmonic forms to prove that a compact oriented n-dimensional Riemannian manifold with positive curvature operators must have the real homology of an n-dimensional sphere [**GM**, Proposition 2.9]. Using the theory of minimal two-spheres, we will outline a proof of the following stronger result.

THEOREM 1. *Let M be a compact simply connected n-dimensional Riemannian manifold with positive curvature operators, where $n \geq 4$. Then M is homeomorphic to a sphere.*

Theorem 1 is actually a consequence of another theorem which makes a weaker hypothesis on the curvature tensor. To describe this hypothesis, we extend the Riemannian metric $\langle\ ,\ \rangle$ in two ways to the complexified tangent space $T_p M \otimes \mathbf{C}$: as a complex symmetric bilinear form $(\ ,\ )$ and as a Hermitian inner product $\langle\langle\ ,\ \rangle\rangle$. Similarly, we extend the metric in two ways to $\Lambda^2 T_p M \otimes \mathbf{C}$. An element $z \in T_p M \otimes \mathbf{C}$ is said to be *isotropic* if $(z,z) = 0$. A complex linear subspace $V \subseteq T_p M \otimes \mathbf{C}$ is *totally isotropic* if $z \in V \Rightarrow (z,z) = 0$.

Finally, we extend the curvature operator $\mathcal{R}$ to a complex linear map $\mathcal{R}: \Lambda^2 T_p M \otimes \mathbf{C} \to \Lambda^2 T_p M \otimes \mathbf{C}$.

DEFINITION. The curvature operator $\mathcal{R}$ is *positive on complex totally isotropic two-planes* if whenever $\{z, w\}$ is a basis for a totally isotropic subspace of $T_p M \otimes \mathbf{C}$ of complex dimension two,

$$\langle\langle \mathcal{R}(z \wedge w), z \wedge w \rangle\rangle > 0.$$

(Note that M has positive sectional curvatures if and only if its curvature operator $\mathcal{R}$ is positive on *real* two-planes.)

By means of a purely algebraic argument, it is possible to prove that if the sectional curvatures $K(\sigma)$ of a Riemannian manifold M of dim ≥ 4 satisfy the inequality $1/4 < K(\sigma) \leq 1$, then the curvature operator of M is positive

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