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AN ALMOST-ORTHOGONALITY PRINCIPLE WITH APPLICATIONS TO MAXIMAL FUNCTIONS ASSOCIATED TO CONVEX BODIES

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1. Introduction. Let B be a convex body in R^n, normalised to have volume one. Let M be the centred Hardy-Littlewood maximal function defined with respect to B, i.e.

$$Mf(x) = \sup_t t^{-n} \int_{tB} |f(x-y)|\, dy.$$

Let $\tilde{M}$ be the lacunary maximal operator,

$$\tilde{M}f(x) = \sup_k 2^{-kn} \int_{2^k B} |f(x-y)|\, dy.$$

Considerable interest has recently been shown in the behaviour of these operators for large n, see [**1, 2, 8, 9, 10**]. When B is the ball, Stein has shown [**8**] that M is bounded on $L^p(R^n)$, $1 < p \leq \infty$, with a constant C_p depending only on p, and not on n; Stein and Strömberg [**10**] have shown that for p larger than 1, the L^p operator norm of M is at most linear in the dimension. More recently Bourgain has proved that the L^2 operator norm of M is bounded by an absolute constant independent of the body and the dimension [**1**]. It is the purpose of this note to extend this result to $p > 3/2$, and to all $p > 1$ if we instead consider $\tilde{M}$.

THEOREM 1. (i) *Let $p > 3/2$. Then there exists a constant C_p, depending only on p and not on B or n, such that* $\|Mf\|_p \leq C_p \|f\|_p$.

(ii) *Let $p > 1$. Then there exists a constant D_p, depending only on p and not on B or n, such that* $\|\tilde{M}f\|_p \leq D_p \|f\|_p$.

It has recently been brought to the author's attention that part (i) of the theorem has been proved by Bourgain[2] in the special case that B is the cube [**2**]. Here we show that Theorem 1 in fact follows from Bourgain's previous analysis together with a general almost-orthogonality principle for maximal functions, Theorem 2. A weaker version of this principle appears in [**6**], where it is also applied to various operators including maximal functions and Hilbert transforms along curves. A similar principle due to Michael Christ appears in [**4**].

Full details of the proofs, together with further applications, will appear elsewhere.

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[2]NOTE ADDED IN PROOF. Theorem 1 has been proved in full independently by J. Bourgain.