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L^p ESTIMATES FOR MAXIMAL FUNCTIONS AND HILBERT TRANSFORMS ALONG FLAT CONVEX CURVES IN $\mathbf{R}^2$

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1. Introduction and statement of results. Let $\Gamma\colon \mathbf{R} \to \mathbf{R}^n$ be a curve in $\mathbf{R}^n$ with $\Gamma(0) = 0$. For suitable test functions f, let $H_\Gamma f(x) = p.v. \int_{-a}^{a} f(x - \Gamma(t))t^{-1}\,dt$ and $M_\Gamma f(x) = \sup_{0<r\leq 1} |r^{-1} \int_0^r f(x - \Gamma(t))\,dt|$. H_Γ and M_Γ are called the Hilbert transform and maximal function along Γ, respectively. There has been considerable interest in estimates of the form $||H_\Gamma f||_p \leq C||f||_p$ and $||M_\Gamma f||_p \leq C||f||_p$ where $||\cdot||_p$ denotes the norm in $L^p(\mathbf{R}^n)$.

If Γ has some curvature at the origin, in a weak sense, then the above L^p estimates for H_Γ and M_Γ have been proved for $1 < p < \infty$ and $1 < p \leq \infty$ respectively, via techniques developed by Nagel, Riviere, Stein, and Wainger; see the survey [**SW**] and the references given there. More recently there has been interest in the case when Γ is flat to infinite order at $t = 0$. In particular if $\Gamma(t) = (t, \gamma(t))$ is a curve in $\mathbf{R}^2$ for which γ is convex for $t > 0$ and either even or odd, then a necessary and sufficient condition for H_Γ to be bounded on L^2 has been obtained in [**NVWW1**]. The condition for odd γ has also turned out to imply the L^2 boundedness of M_Γ [**NVWW2**]. There has also been progress in the study of L^p boundedness for $p \neq 2$ [**NW, CNVWW, C**].

In the present paper we consider (locally) C^1 curves $\Gamma(t) = (t, \gamma(t))$ in $\mathbf{R}^2$ defined for $t \geq 0$, with $\gamma'(0) = \gamma(0) = 0$, convex and increasing. To discuss the Hilbert transform $\Gamma(t)$ must be defined for $t < 0$; we define $\Gamma_e(t) = (t, \gamma(-t))$ and $\Gamma_0(t) = (t, -\gamma(-t))$ for $t < 0$. Curvature hypotheses are replaced by the much weaker "doubling property"

$$\text{there exists } \lambda > 1 \text{ with } \gamma'(\lambda t) \geq 2\gamma'(t) \text{ for all } t > 0. \tag{1.1}$$

We shall prove

THEOREM. *Let $\Gamma, \Gamma_e, \Gamma_0$ be as above and satisfy* (1.1). *Then $||M_\Gamma f||_p \leq C||f||_p$ for $1 < p \leq \infty$, and $||H_{\Gamma_e} f||_p + ||H_{\Gamma_0} f||_p \leq C||f||_p$ for $1 < p < \infty$. More precisely, the latter assertion is that the operators H_Γ, initially defined only for test functions, extend to bounded operators on L^p.*

By combining this theorem with the necessary condition for L^2 boundedness of H_{Γ_e} in [**NVWW1**], we obtain the following

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