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HYPERBOLIC AND DIOPHANTINE ANALYSIS

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Dedicated to F. Hirzebruch

In this survey we consider Kobayashi hyperbolicity, in which there is an interplay between five notions:

analytic notions of distance and measure;
complex analytic notions;
differential geometric notions of curvature (Chern and Ricci form);
algebraic notions of "general type" (pseudo ampleness);
arithmetic notions of rational points (existence of sections).

I am especially interested in the relations of the first four notions with diophantine geometry, which historically has intermingled with complex differential geometry. One of the main points of this survey is to arrive at a certain number of conjectures in an attempt to describe at least some of these relations coherently.

Throughout this article, by an **algebraic set** we mean the set of zeros of a finite number of homogeneous polynomials

$$P_j(x_0, \ldots, x_N) = 0, \qquad j = 1, \ldots, m$$

in projective space $\mathbf{P}^N$ over $\mathbf{C}$, so $x_0, \ldots, x_N$ are the projective coordinates. An algebraic set will be called a **variety** if it is irreducible—that is, the polynomials can be chosen so that they generate a prime ideal. If X has no singularities, then X is also a complex manifold. In any case, X has a complex structure. From now on, we let X denote a variety.

Among the possible complex analytic properties of X we shall emphasize that of being **hyperbolic**. There are several equivalent definitions of this notion, and one of them, due to Brody, is that every holomorphic map of $\mathbf{C}$ into X is constant. At the beginning of this article, we shall give three possible characterizations, including Kobayashi's original definition, and prove the equivalence between them, following Brody.

On the other hand, X also has an algebraic structure. For one thing, taking the algebraic subsets of X as closed subsets defines the Zariski topology. Thus the **Zariski closure** of a set S is the smallest algebraic set containing S, and is equal to the intersection of all algebraic sets containing S. Furthermore, the polynomials P_j have coefficients in some field F_0, finitely generated over the rational numbers, and this gives rise to diophantine properties as follows.

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