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Computation with recurrence relations, by Jet Wimp, Applicable Mathematics Series, Pitman Advanced Publishing Program, Boston, London, Melbourne, 1984, xii + 310 pp., $50.00. ISBN 0-273-08508-5

Recurrence relations occur in a variety of mathematical contexts. They connect a set of elements of a sequence of some type, usually either numbers or functions, such as coefficients in series expansions obtained by undetermined coefficients, moments of weight functions, and members of families of special functions. They can be used either to define the sequence or to produce its elements.

They lead to concise algorithms which are useful for either manual or automatic calculations and can allow great economy in tabulation or approximation. Algorithms based on recurrences are particularly useful for automatic computers because of the compact programs to which they lead, with concomitant economies in memory requirements and in error elimination.

Serious difficulties may be encountered, however, when inexact arithmetic or initial values are used. For example, the modified Bessel functions of the first kind, $I_n(x)$ satisfy the recurrence:

$$y_{n+1}(x) = -(2n/x)y_n(x) + y_{n-1}(x). \tag{1}$$

For $x = 1$, they are positive for all n, and decrease monotonously toward 0 as n increases. Using values for $I_0(1) = 1.266065878$ and $I_1(1) = 0.5651591040$, correct to 10 significant digits, and computing $I_2(1), I_3(1), \ldots$ by (1), we find

n	$I_n(1)$	n	$I_n(1)$	n	$I_n(1)$
0	0.1266065878 (+1)	1	0.5651591040 (00)	2	0.1357476700 (00)
3	0.2216842400 (−1)	4	0.2737126000 (−2)	5	0.2714160000 (−3)
6	0.2296600000 (−4)	7	−0.4176000000 (−5)	8	0.8143000000 (−4)
9	−0.1307056000 (−2)	10	0.2360843800 (−1)	11	−0.4734758160 (00)
12	0.1044007639 (+2)	13	−0.2510353092 (+3)	14	0.6537358115 (+4)

These absurd numerical values are caused by instability in using this recurrence for $I_n(x)$ for increasing n. Such difficulties are familiar to numerical mathematicians in many contexts, although they may not be as generally recognized as would be desirable.