BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 1, January 1986

0273-0979/86 $1.00 + $.25 per page

The H-functions of one and two variables with applications, by H. M. Srivastava, K. C. Gupta, and S. P. Goyal, South Asian Publishers, New Delhi, India, 1982, x + 306 pp., $25.00.

The generalized hypergeometric function ${}_pF_q$ is defined by the power series

$$(1)\qquad {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x)=\sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_p)_n}{(b_1)_n\cdots(b_q)_n}\frac{x^n}{n!}$$

where $(\alpha)_\mu=\Gamma(\alpha+\mu)/\Gamma(\alpha)$ is the Pochhammer symbol. We call x the *variable*, while $a_1,\ldots,a_p$, $b_1,\ldots,b_q$ are the *parameters*. Among the particular cases of ${}_pF_q$ are Gauss's hypergeometric function ${}_2F_1$ and the confluent hypergeometric function (Kummer's function) ${}_1F_1$.

The condition $p\leqslant q+1$ is necessary to ensure convergence in (1) for $x\neq 0$ (unless the series is terminating). The differential equation for ${}_pF_q$, however, makes sense without this restriction; and in fact one can, by considering suitable integral representations, construct functions that might be termed extensions of generalized hypergeometric functions to the case $p>q+1$. One such function is MacRobert's E-function from 1937, defined by an integral over $\mathbf{R}_+^p$. More successful, however, are subsequent generalizations in terms of single Mellin-Barnes integrals: Meijer's G-function from 1941 and the even more general H-function introduced by Fox in 1961. The latter is defined as follows,

$$(2)\qquad \begin{cases} H_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_1,\alpha_1),\ldots,(a_p,\alpha_p)\\(b_1,\beta_1),\ldots,(b_q,\beta_q)\end{matrix}\right.\right]=\dfrac{1}{2\pi i}\displaystyle\int_L\theta(s)x^s\,ds,\\[2ex] \theta(s)=\dfrac{\prod_{j=1}^m\Gamma(b_j-\beta_js)\prod_{j=1}^n\Gamma(1-a_j+\alpha_js)}{\prod_{j=m+1}^q\Gamma(1-b_j+\beta_js)\prod_{j=n+1}^p\Gamma(a_j-\alpha_js)},\end{cases}$$

where L is a suitable contour in the complex plane. $H_{p,q}^{m,n}$ reduces to a Meijer function when all α's and β's are equated to unity. It is noted that Fox's function has *four* types of parameters. Meijer's and Fox's functions are treated in detail in two recent monographs by Mathai and Saxena [**1**, **2**].