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NEW RESULTS FOR COVERING SYSTEMS OF RESIDUE SETS

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We announce some new results about systems of residue sets. A residue set $R \subset \mathbf{Z}$ is an arithmetic progression

$$R = \{a, a \pm n, a \pm 2n, \ldots\}.$$

The positive integer n is referred to as the *modulus* of R. Following Znám [**21**] we denote this set by $a(n)$. We need several number-theoretic functions.

$p(m)$–the least prime divisor of a natural number m,

$P(m)$–the greatest prime divisor of m,

$\Lambda(m)$–the greatest divisor of m which is a power of a single prime:

$$\Lambda(m) = \max\{d \in \mathbf{Z} \colon d|m, d = p^s, \quad p \textit{ prime}\},$$

$f(m) = \sum_{j=1}^{l} s_j(p_j - 1) + 1$, where m has the prime factorization $m = p_1^{s_1} \cdots p_l^{s_l}$,

$g(m) = \prod_{j=1}^{l}(1 + x_j) - \sum_{j=1}^{l} x_j - 1$, where

$$x_j = \frac{\sum_{k=0}^{s_j-1} p_j^k}{p_j^{s_j} - \sum_{k=0}^{s_j-1} p_j^k}$$

and m has the above prime factorization,

$\varphi(m)$–Euler's totient function,

$[x]$–the greatest integer in x.

Recent general surveys on systems of residue sets are Porubský [**21**] and Znám [**26**]. Results and problems on residue sets appear also in Erdős and Graham [**14**] and Guy [**16**].

1. Disjoint covering systems [1, 2, 3, 6, 9, 10]. These are systems $\mathcal{D} = (a_1(n_1), \ldots, a_t(n_t))$, $t > 1$, which partition $\mathbf{Z}$. The multiplicity of a modulus $n = n_k$ is the number of sets in $\mathcal{D}$ with that modulus. The multiplicity of $\mathcal{D}$ is the maximum multiplicity of its moduli.

THEOREM 1. *The multiplicity of any modulus $n = n_k$ is at least*

$$m_1 = \min_{n_i \neq n} \Lambda\left(\frac{n}{(n, n_i)}\right). \tag{1}$$

The multiplicity of $\mathcal{D}$ is at least

$$m_2 = \left[\frac{P(N)\varphi(N)}{N}\right] + 1, \tag{2}$$

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