BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 13, Number 2, October 1985

ON PLATEAU'S PROBLEM FOR MINIMAL SURFACES OF HIGHER GENUS IN $\mathbf{R}^3$

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The classical solution of the Plateau problem by Radó [**10**] and Douglas [**3**] shows that any rectifiable Jordan curve in $\mathbf{R}^3$ is spanned by a minimal surface of disc type. Under what conditions a minimal surface of a given higher genus exists, spanning a given Jordan curve in a Riemannian manifold N, seems to be a much more difficult problem. For compact minimal surfaces without boundary and in case N has sufficient topological complexity, the "incompressibility" method of Schoen and Yau gives a sufficient condition for existence.

In [**4**] Douglas did develop a method to treat the problem of when a *given* contour is spanned by a surface of genus $\mathfrak{p}$. Douglas' condition, however, seems quite difficult to verify in concrete cases. In this paper we will give simple geometric and topological sufficient conditions.

THEOREM. *Let N be a solid torus of class C^3 and genus $\mathfrak{g}$ in $\mathbf{R}^3$ whose boundary has nonnegative inward mean curvature, and let $\gamma \in \Pi_1(N)$ denote the homotopy class of a rectifiable Jordan curve Γ in N.*

(a) *If $\mathfrak{g} = 2\mathfrak{p}$ and $\gamma = a_1 a_2 a_1^{-1} a_2^{-1} \cdots a_{2\mathfrak{p}-1} a_{2\mathfrak{p}} a_{2\mathfrak{p}-1}^{-1} a_{2\mathfrak{p}}^{-1}$ where $a_1, \ldots, a_{2\mathfrak{p}}$ is a basis for $\Pi_1(N)$ then Γ bounds an immersed oriented minimal surface of genus $\mathfrak{p}$.*

(b) *If $\mathfrak{g} = 1$ and $\gamma = 2\alpha$ for some $\alpha \neq 0$ in $\Pi_1(N)$ then Γ bounds an immersed minimal surface of Möbius type.*

We sketch the proof of part (a).

Let Γ be a rectifiable contour in a solid $2\mathfrak{p}$ torus $N \subset \mathbf{R}^3$, M a surface of genus $\mathfrak{p}$ with $\partial M \cong S^1$ the unit circle. Further let $\mathcal{N}_\Gamma = \{u\colon M \to N \mid u\colon M \to \Gamma$ monotonically, $u \in H^1(M, \mathbf{R}^3) \cap C(M, \mathbf{R}^3)\}$. Denote by $\mathcal{M}$ the C^∞ Riemannian metrics g on the Schottky double $\hat{M}$ of M such that the natural involution $T\colon \hat{M} \hookleftarrow$ is an isometry for g. Dirichlet's functional

$$E\colon \mathcal{M} \times \mathcal{N}_\Gamma \to R$$

is defined by

$$E(g,u) = \frac{1}{2} \sum_{i=1}^{3} \int_M g(x)(\nabla_g u^i, \nabla_g u^i)\, d\mu_g.$$

Let P be the space of all C^∞ positive functions on $\hat{M}$ which are symmetric and D_0 those C^∞ diffeomorphisms which fix $\partial M \subset \hat{M}$ (as a set) and are homotopic to the identity. The Teichmüller space for M is then defined to be $\mathcal{T} = (\mathcal{M}/P)/D_0$, a finite-dimensional C^∞ manifold of dimension

Received by the editors May 13, 1985.
1980 *Mathematics Subject Classification*. Primary 53A10, 49F10.