

ON THE EFFICIENCY OF ALGORITHMS OF ANALYSIS

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CHAPTER I

1. Introduction. This is an expanded version of the Jaqueline Lewis talks I gave at Rutgers in April 1984. It is partly an exposition of recent results and new open problems. Also, some new proofs are given here. The subject is the global analysis of algorithms of linear and calculus mathematics, especially in regard to efficiency. This is part of the subject called computational complexity. However, in the past, computational complexity has usually referred to the study of algorithms for discrete problems. In what follows, the problems come from numerical analysis, operations research, and classical mathematics ("continuous" classical mathematics). It is sometimes forgotten how close numerical analysis and classical mathematics are to each other. But to confirm this relationship one can note the frequent appearance of the names of Newton, Lagrange, Gauss in numerical analysis texts, and look at *Goldstine*.

Received by the editors February 7, 1985.

1980 *Mathematics Subject Classification*. Primary 65-02.

¹I would like to thank N. H. Kuiper and IHES for their hospitality. I would like also to acknowledge NSF support during the preparation of this paper.

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0273-0979/85 \$1.00 + \$.25 per page