

# RESEARCH ANNOUNCEMENTS

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## UNITARY DUAL OF $p$ -ADIC $GL(n)$ . PROOF OF BERNSTEIN CONJECTURES

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**1. Introduction.** A fundamental problem of harmonic analysis on a locally compact group  $G$  is the description of the dual object  $\hat{G}$  of  $G$ , i.e. description of the set of all equivalence classes of irreducible unitary representations of  $G$ . If  $G$  is a connected reductive  $p$ -adic group then  $\hat{G}$  is in the natural bijection with the subset of all unitarizable classes in the set  $\tilde{G}$  consisting of all equivalence classes of irreducible smooth representations. In this way the problem of parametrizing  $\hat{G}$  breaks into two problems, the problem of describing the nonunitary dual  $\tilde{G}$  and the problem of identifying unitarizable classes in  $\tilde{G}$ . The first problem has been studied much more than the second one, which is solved completely only for groups  $SL(2)$ ,  $GL(2)$ . In the case of reductive Lie groups, the second problem has been solved only for some groups of low ranks. This paper announces the complete solution of the second problem for the case of the general linear group  $GL(n)$  over a  $p$ -adic field  $F$  of characteristic zero (we describe Langlands parameters of  $GL(n, F)^\wedge$ ). Proof of Bernstein conjectures on unitarizability in [1] is also announced.

**2. Main results.** Let  $R_n$  be the Grothendieck group of the category of all smooth representations of  $GL(n, F)$  of finite length. We consider  $GL(n, F)^\sim \subseteq R_n$ . Set  $R = \bigoplus R_n$  ( $n \geq 0$ ),  $\text{Irr} = \text{UGL}(n, F)^\sim$  ( $n \geq 0$ ) and  $\text{Irr}^u = \text{UGL}(n, F)^\wedge$  ( $n \geq 0$ ). The induction functor  $(\tau, \sigma) \mapsto \tau \times \sigma$  defines a structure of the commutative graded ring on  $R$  (see [8]). Let  $D_0$  be the subset of all square integrable representations in  $\text{Irr}$ . The character  $g \mapsto |\det g|$  of  $GL(n, F)$  is denoted by  $\nu$ . Set  $D = \{\nu^\alpha \delta; \alpha \in \mathbf{R}, \delta \in D_0\}$ . The set of all finite multisets

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