A NEW POLYNOMIAL INVARIANT OF KNOTS AND LINKS[1]

BY P. FREYD, D. YETTER; J. HOSTE;
W. B. R. LICKORISH, K. MILLETT; AND A. OCNEANU

The purpose of this note is to announce a new isotopy invariant of oriented links of tamely embedded circles in 3-space.

We represent links by plane projections, using the customary conventions that the image of the link is a union of transversely intersecting immersed curves, each provided with an orientation, and undercrossings are indicated by broken lines. Following Conway [6], we use the symbols L_+, L_0, L_- to denote links having plane projections which agree except in a small disk, and inside that disk are represented by the pictures of Figure 1.

Conway showed that the one-variable Alexander polynomials of L_+, L_0, L_- (when suitably normalized) satisfy the relation

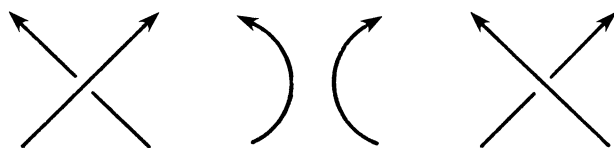$$\Delta_{L_+}(t) - \Delta_{L_-}(t) + (t^{1/2} - t^{-1/2})\Delta_{L_0}(t) = 0.$$

FIGURE 1

[1] *Editor's Note.* The editors received, virtually within a period of a few days in late September and early October 1984, four research announcements, each describing the same result—the existence and properties of a new polynomial invariant for knots and links. There was variation in the approaches taken by the four groups and variation in corollaries and elaboration. These were: *A new invariant for knots and links* by Peter Freyd and David Yetter; *A polynomial invariant of knots and links* by Jim Hoste; *Topological invariants of knots and links*, by W. B. R. Lickorish and Kenneth C. Millett, and *A polynomial invariant for knots: A combinatorial and an algebraic approach*, by A. Ocneanu.

It was evident from the circumstances that the four groups arrived at their results completely independently of each other, although all were inspired by the work of Jones (cf. [10], and also [8, 9]). The degree of simultaneity was such that, by common consent, it was unproductive to try to assess priority. Indeed it would seem that there is enough credit for all to share in.

Each of these papers was refereed, and we would have happily published any one of them, had it been the only one under consideration. Because the alternatives of publication of all four or of none were both unsatisfying, all have agreed to the compromise embodied here of a paper carrying all six names as coauthors, consisting of an introductory section describing the basics written by a disinterested party, and followed by four sections, one written by each of the four groups, briefly describing the highlights of their own approach and elaboration.