

BOOK REVIEWS

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Intersection theory, by William Fulton, *Ergebnisse der Mathematik und ihrer Grenzgebiete 3. Folge · Band 2*, Springer-Verlag, 1984, xi + 470 pp., \$39.00. ISBN 3-5401-2176-5

Introduction to intersection theory in algebraic geometry, by William Fulton, *Regional Conference Series in Mathematics*, Vol. 54, American Mathematical Society, 1984, v + 82 pp., \$16.00. ISBN 0-8218-0704-8

Intersection theory is an old and basic part of algebraic geometry. Algebraic geometry is the mathematics of loci defined by algebraic (polynomial) equations; currently such loci are called schemes. Intersection theory concerns the intersection of two schemes meeting in a third; in other words [7], intersection theory is “the system of assumptions, accepted principles, and rules of procedure devised to analyze, predict, or otherwise explain the nature or behavior of” such an intersection.

The two books under review are complete up-to-date accounts of intersection theory. The *Ergebnisse* book offers a detailed treatment; the CBMS book, a general introduction. Both present a revolutionary new approach, developed by the author in collaboration with MacPherson, which is technically simpler and cleaner, yet much more refined and general. To better appreciate the subject of intersection theory and the contribution of these books, it is useful to know some history.

Intersection theory was founded in 1720 by Maclaurin, 93 years after Descartes promoted the use of coordinates and equations [6, pp. 552–554, 607–608]. Maclaurin stated that two curves, defined by equations of degrees m and n , intersect in mn points. A proof was sought by Euler in 1748 and Cramer in 1750. Finally in 1764 Bezout introduced a more refined method of eliminating one of the two variables from the two equations, producing a polynomial in one variable of minimal degree, which he proved is equal to mn . (Euler did so independently the same year.) Bezout went on to treat the case of r equations in r unknowns [1, p. 292], and so most any theorem of intersection theory about projective r -space is called Bezout’s theorem.

Intersection theory remained centered around Bezout’s theorem for a century and a half. Maclaurin used it to show that an irreducible curve of degree n has at most $(n - 1)(n - 2)/2$ singular points. Maclaurin and others used it to determine the degree of geometrically defined loci. Goudin and du Séjour in