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A POLYNOMIAL INVARIANT FOR KNOTS VIA VON NEUMANN ALGEBRAS[1]

BY VAUGHAN F. R. JONES[2]

A theorem of J. Alexander [**1**] asserts that any tame oriented link in 3-space may be represented by a pair (b, n), where b is an element of the n-string braid group B_n. The link L is obtained by closing b, i.e., tying the top end of each string to the same position on the bottom of the braid as shown in Figure 1. The closed braid will be denoted $b^\wedge$.

Thus, the trivial link with n components is represented by the pair $(1, n)$, and the unknot is represented by $(s_1 s_2 \cdots s_{n-1}, n)$ for any n, where $s_1, s_2, \ldots, s_{n-1}$ are the usual generators for B_n.

The second example shows that the correspondence of (b, n) with $b^\wedge$ is many-to-one, and a theorem of A. Markov [**15**] answers, in theory, the question of when two braids represent the same link. A Markov move of type 1 is the replacement of (b, n) by (gbg^{-1}, n) for any element g in B_n, and a Markov move of type 2 is the replacement of (b, n) by $(bs_n^{\pm 1}, n+1)$. Markov's theorem asserts that (b, n) and (c, m) represent the same closed braid (up to link isotopy) if and only if they are equivalent for the equivalence relation generated by Markov moves of types 1 and 2 on the *disjoint* union of the braid groups. Unforunately, although the conjugacy problem has been solved by F. Garside [**8**] within each braid group, there is no known algorithm to decide when (b, n) and (c, m) are equivalent. For a proof of Markov's theorem see J. Birman's book [**4**].

The difficulty of applying Markov's theorem has made it difficult to use braids to study links. The main evidence that they might be useful was the existence of a representation of dimension $n - 1$ of B_n discovered by W. Burau in [**5**]. The representation has a parameter t, and it turns out that the determinant of $1-$(Burau matrix) gives the Alexander polynomial of the closed braid. Even so, the Alexander polynomial occurs with a normalization which seemed difficult to predict.

In this note we introduce a polynomial invariant for tame oriented links via certain representations of the braid group. That the invariant depends only on the closed braid is a direct consequence of Markov's theorem and a certain trace formula, which was discovered because of the uniqueness of the trace on certain von Neumann algebras called type II_1 factors.

Notation. In this paper the Alexander polynomial Δ will always be normalized so that it is symmetric in t and t^{-1} and satisfies $\Delta(1) = 1$ as in Conway's tables in [**6**].

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[2]The author is a Sloan foundation fellow.