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LOCAL MODULI FOR MEROMORPHIC DIFFERENTIAL EQUATIONS

BY DONALD G. BABBITT AND V. S. VARADARAJAN

1. Introduction. This note announces results concerning the parametrization, in the sense of (local) moduli, of the equivalence classes of systems of meromorphic differential equations of the form

$$(*) \qquad du/dz = Au$$

near an irregular singular point (assumed to be $z = 0$). Here u is an n-component column vector, A is an $n \times n$ matrix of meromorphic functions, and equivalence of systems defined by matrices A and B means that there is a meromorphic invertible $n \times n$ matrix x such that

$$(**) \qquad x[A] \stackrel{\text{def}}{=} xAx^{-1} + (dx/dz)x^{-1} = B$$

near $z = 0$. If $\mathcal{F}_{\text{cgt}}$ (resp. $\mathcal{F}$) is the field of quotients of the ring of convergent (resp. formal) power series in z with coefficients in $\mathbf{C}$, $(**)$ defines an action of $\mathrm{GL}(n, \mathcal{F}_{\text{cgt}})$ on $\mathfrak{gl}(n, \mathcal{F}_{\text{cgt}})$, reflecting the fact that $(*)$ goes over to the system $dv/dz = Bv$ under the substitution $v = xu$; replacing $\mathcal{F}_{\text{cgt}}$ by $\mathcal{F}$ leads to the notion of formal equivalence. We note that for any commutative ring R (with unit) equipped with a derivation D, $(**)$ defines an action of $\mathrm{GL}(n, R)$ on $\mathfrak{gl}(n, R)$, with D replacing d/dz; if R is a suitably restricted ring of Laurent series in z with coefficients in the ring of convergent power series in d variables and $D = d/dz$, we obtain the notion of equivalence of analytic families of systems $(*)$ depending on d parameters, which is basic to the theory of local moduli (cf. [**BV2**]).

One parametrizes the equivalence classes of systems $(*)$ in two steps. The first step is the classification up to formal equivalence, i.e., the description of the orbit space $\mathrm{GL}(n, \mathcal{F})\backslash\mathfrak{gl}(n, \mathcal{F})$; the second step is to fix a formal class Ω with $\Omega_{\text{cgt}} \stackrel{\text{def}}{=} \Omega \cap \mathfrak{gl}(n, \mathcal{F}_{\text{cgt}}) \neq \emptyset$, and to classify the systems $(*)$ in Ω_{cgt} up to equivalence, i.e., to describe the orbit space $\mathrm{GL}(n, \mathcal{F}_{\text{cgt}})\backslash\Omega_{\text{cgt}}$. The description of $\mathrm{GL}(n, \mathcal{F})\backslash\mathfrak{gl}(n, \mathcal{F})$; goes back to Hukuhara and Turrittin (see [**BV1**] for extensive references) and is based on the notion of a canonical form. The classical method of studying the second question is based on the technique of Stokes lines and Stokes multipliers [**Bi**, **J**]. Recently this has been examined from a more modern, and essentially cohomological, point of view, notably by Malgrange [**Ma1**, **Ma2**], Sibuya [**S**], and Deligne (cf. [**Be**]). The present note continues this theme by studying the equivalence of analytic families of systems $(*)$ and is based in a fundamental way on the theory of formal equivalence over general rings developed in [**BV2**].

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