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CANONICAL PERTURBATION THEORY OF ANOSOV SYSTEMS, AND REGULARITY RESULTS FOR THE LIVSIC COHOMOLOGY EQUATION

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We have studied the following problem.

Given a compact symplectic manifold (M, ω) and a C^∞ family of C^∞ symplectic transformations f_ε, when is it true that there exists another family of C^∞ diffeomorphisms g_ε such that

$$(*) \qquad g_\varepsilon \circ f_0 = f_\varepsilon \circ g_\varepsilon?$$

The motivation, of course, is canonical perturbation theory, where f_0 is supposed to be "well understood", and we want to use this knowledge to get information about similar systems. Typically, f_0 is an integrable system, but there are nonintegrable systems whose behaviour is also known in great detail, and one could consider perturbation theories for them. Indeed, particular perturbation theories have been considered for geodesic flows in surfaces or manifolds of negative curvature [**G.K.2.3**; **C.E.G**].

Our results show that there is a very satisfactory answer when f_0—and hence f_ε for small ε—have some hyperbolicity in them.

DEFINITION. We say f_ε is a globally hamiltonian isotopy (G.H.I.) when

$$df_\varepsilon/d\varepsilon = \tilde{F}_\varepsilon \circ f_\varepsilon,$$

$\tilde{F}_\varepsilon$ being a globally hamiltonian vector field of hamiltonian F_ε with vanishing average (similarly for local hamiltonian isotopies, L.H.I.).

Since F_ε determines f_ε, if we only look for g_ε which are G.H.I. starting at the identity, $(*)$ is equivalent to

$$(**) \qquad F_\varepsilon = G_\varepsilon - G_\varepsilon \circ f_\varepsilon^{-1},$$

This is roughly equivalent to the use of generating functions in classical theory to rewrite equations between diffeomorphisms as equations between functions. Generating functions cannot be defined in all manifolds, whereas

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