FIVE SHORT STORIES ABOUT THE CARDINAL SERIES

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INTRODUCTION

Suppose that a function g generates a Fourier series in the usual way:

\[ g(x) \sim \sum c_n e^{-inx}. \]

Now multiply both members by \( e^{ixt}/2\pi \) and formally integrate over a period. On the right we obtain

\[ \sum c_n \frac{\sin \pi(t-n)}{\pi(t-n)}, \]

or, equivalently,

\[ \frac{\sin \pi t}{\pi} \sum c_n \frac{(-1)^n}{t-n}, \]

which is called a "cardinal series". On the left we obtain a function \( f \) whose form

\[ f(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} g(x) e^{ixt} \, dx \]

suggests that it has a Fourier transform with compact support on \([-\pi, \pi]\), or, put another way, \( f \) has no frequency content outside the "band" \([-\pi, \pi]\). One can expect that such an \( f \) will be represented in some sense by the cardinal series (1), and that in all likelihood the coefficient \( c_n \) will, because of (2), be of the form \( f(n) \).

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