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FIVE SHORT STORIES ABOUT THE CARDINAL SERIES

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INTRODUCTION

Suppose that a function g generates a Fourier series in the usual way:

$$g(x) \sim \sum c_n e^{-inx}.$$

Now multiply both members by $e^{ixt}/2\pi$ and formally integrate over a period. On the right we obtain

$$\sum c_n \frac{\sin \pi(t-n)}{\pi(t-n)}, \tag{1a}$$

or, equivalently,

$$\frac{\sin \pi t}{\pi} \sum c_n \frac{(-1)^n}{t-n}, \tag{1b}$$

which is called a "cardinal series". On the left we obtain a function f whose form

$$f(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(x) e^{ixt}\, dx \tag{2}$$

suggests that it has a Fourier transform with compact support on $[-\pi, \pi]$, or, put another way, f has no frequency content outside the "band" $[-\pi, \pi]$. One can expect that such an f will be represented in some sense by the cardinal series (1), and that in all likelihood the coefficient c_n will, because of (2), be of the form $f(n)$.

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