BOOK REVIEWS

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Stone spaces, by Peter T. Johnstone, Cambridge Studies in Advanced Mathematics, Vol. 3, Cambridge University Press, New York, 1983, xxi + 370 pp., $59.50. ISBN 0-5212-3983-5

This book is a bid to create Stone Spaces, much as Genghis Khan created Mongolia. Not, of course, from the void. Stone theory has been driven by the dialectic between spaces and rings or lattices. It begins in Boolean algebras, which are rings and lattices at once. It is their dual objects, the totally disconnected compact Hausdorff spaces, that are to be called Stone spaces.

(Better term, if the only obstruction were the usual name, Boolean spaces; but what is to be done with the established stonian and hyperstonian spaces?)

In this area, natural and nonnatural constructions combine in fairly complex ways. First half of a notable illustration: In Boolean algebras as in abelian groups there are injective envelopes (in the now standard sense), given by MacNeille completion, and practically identified by Sikorski [**20**] as injective envelopes. Hence, by mere duality, Boolean (or Stone) spaces have projective covers. But Gleason found [**6**] that all compact Hausdorff spaces have projective covers: the same (Boolean) projectives, and constructed by something very like MacNeille completion. "The same projectives" is easily explained (and soon was [**18**]): the compact Hausdorff spaces form what is now familiar as the Eilenberg-Moore category of a monad, and projectives are just retracts of free spaces. Boolean spaces live on the same monad and form the smallest quasivariety containing its free spaces. Explaining "something very like" MacNeille completion takes us much further. The Gleason cover of compact Hausdorff X is reached by way of (1) the lattice of open sets ΩX, and (2) the quotient lattice $(\Omega X)_{\neg\neg}$ in which elements u, v are identified if $t \wedge u = \varnothing \Leftrightarrow t \wedge v = \varnothing$, by observing that $(\Omega X)_{\neg\neg}$ is (though not a topology) a Boolean algebra, and taking its dual. Since the crucial station (2) is not a topology, traveling through it requires keeping an eye on three categories at once, or inventing (as in fact Ehresmann did [**3**] before Gleason's theorem) the category of pointless topological spaces or *locales*, and describing the Gleason cover of the spatial locale X as the (spatial) compact Hausdorff reflection of the (nonspatial) smallest dense sublocale $D(X)$. Obviously this is not a reduction to something simpler, but it is an explanation of another frequently useful type, surrounding a beautiful construction with a theory.

Gleason's theorem might be called the Kashgar of Stoneland; it leads to further developments in topological spaces and in toposes. But the second half