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NEW RESULTS ON THE AVERAGE BEHAVIOR OF SIMPLEX ALGORITHMS[1]

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ABSTRACT. It has been a challenge for mathematicians to theoretically confirm the extremely good performance of simplex algorithms for linear programming. We have confirmed that a certain variant of the simplex method solves problems of order $m \times n$ in an expected number of steps which is bounded between two quadratic functions of the *smaller* dimension of the problem. Our probabilistic assumptions are rather weak.

1. Introduction. We consider the linear programming problem of order $m \times n$ in the form: Maximize $c^T x$ over all $x \geq 0$ in R^n such that $Ax \leq b$, where $A \in R^{m \times n}$. It has been observed that the simplex algorithms for linear programming, developed by George Dantzig [**D**], work extremely well. A central question in the field of analysis of algorithms is to estimate the expected number of steps that these algorithms perform relative to different probability distributions of inputs. Some background is given in §2. In [**AM1** and **T**] we consider a variant of the simplex method called the "lexicographic self-dual method". We find that the expected number of steps of this variant, relative to a rather weak probabilistic model, is only $O((\min(m,n))^2)$. This is the first polynomial average-case upper bound for a simplex algorithm which is capable of solving any linear programming problem. The first two authors [**AM2**] have also determined a quadratic lower bound, so the behavior of this variant is indeed quadratic, whereas it has been conjectured that other variants take a linear expected number of steps. With various choices of the starting point, this variant can simulate different "constraint-by-constraint" and "variable dimension" algorithms, as well as combinations thereof [**Me2**]. Adler, Karp and Shamir [**AKS**] provide a different proof of similar upper bound for a certain class of "constraint-by-constraint" algorithms. It follows from [**Me2**] that their variant can also be simulated by the self-dual algorithm with an appropriate starting point.

2. Background. The simplex algorithms iteratively change the basis of a linear system of equations, until they reach an "optimal" basis, or a basis that exhibits that no optimal solution exists. Potentially, there are $\binom{m+n}{m}$

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