

COMPLEX ANALYTIC DYNAMICS ON THE RIEMANN SPHERE

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Table of Contents

1. Background and notation
 2. The dynamical dichotomy of Fatou and Julia
 3. Periodic points
 4. The consequences of Montel's Theorem
 5. The Julia set is the closure of the set of repelling periodic points
 6. Classical results concerning the Fatou set
 7. Sullivan's classification of the Fatou set
 8. A condition for expansion on the Julia set
 9. The dynamics of polynomials
 10. The Mandelbrot set and the work of Douady and Hubbard
 11. The measurable Riemann mapping theorem and analytic dynamics
 12. Bibliographic notes
- List of notation
References

Holomorphic, noninvertible dynamical systems of the Riemann sphere are surprisingly intricate and beautiful. Often the indecomposable, completely invariant sets are fractals (à la Mandelbrot [M1]) because, in fact, they are quasi-self-similar (see Sullivan [S3] and (8.5)). Sometimes they are nowhere differentiable Jordan curves whose Hausdorff dimension is greater than one (Sullivan [S4] and Ruelle [R]). Yet these sets are determined by a *single* analytic function $z_{n+1} = R(z_n)$ of a *single* complex variable.

The study of this subject began during the First World War. Both P. Fatou and G. Julia independently published a number of *Compte Rendu* notes, and then both wrote long *memoires*—Julia [J] in 1918 and Fatou [F1–F3] in 1919 and 1920. At that time, they had at their disposal a new theorem of Montel (see (4.1)) which gave a sufficient condition for the normality of a family of meromorphic functions. They applied the theory of normal families to the dynamical system to prove some remarkable results.

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