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Group extensions, representations, and the Schur multiplicator, by F. Rudolf Beyl and Jürgen Tappe, Lecture Notes in Math., Vol. 958, Springer-Verlag, Berlin, 1982, iv + 278 pp., $13.50. ISBN 3-5401-1954-X

Schur multipliers arise when one studies central extensions of groups. A central extension is a surjective homomorphism $\varphi\colon G \to Q$ whose kernel is contained in the center of G. One also calls G itself a central extension of Q. Schur was interested in finding all projective representations of a given finite group Q, i.e. all homomorphisms $\rho\colon Q \to \mathrm{PGL}_n(\mathbf{C})$ with $n \geqslant 2$. The group $\mathrm{PGL}_n(\mathbf{C})$ comes with a central extension $\pi\colon \mathrm{GL}_n(\mathbf{C}) \to \mathrm{PGL}_n(\mathbf{C})$, where π is the usual map associating with a linear transformation of $\mathbf{C}^n$ an automorphism of projective $n - 1$ space ($n \geqslant 2$). The kernel of π is the center of $\mathrm{GL}_n(\mathbf{C})$ and may be identified with $\mathbf{C}^* = \mathrm{GL}_1(\mathbf{C})$. Pulling back π along ρ one gets a central extension $\varphi\colon G \to Q$ with kernel $\mathbf{C}^*$ and the situation is that of Diagram 1.

$$\begin{array}{ccc} G & \overset{\varphi}{\to} & Q \\ \downarrow \sigma & & \downarrow \rho \\ \mathrm{GL}_n(\mathbf{C}) & \overset{\pi}{\to} & \mathrm{PGL}_n(\mathbf{C}) \end{array}$$

DIAGRAM 1

Thus we have associated with the projective representation ρ of Q the linear representation σ of G. Conversely, if $\varphi\colon G \to Q$ is any central extension and $\sigma\colon G \to \mathrm{GL}_n(\mathbf{C})$ is an irreducible linear representation, one obtains by Schur's Lemma a projective representation ρ of Q such that Diagram 1 commutes. Schur discovered [7, 1902] that there is at least one finite central extension $\varphi\colon G \to Q$ such that σ exists for all ρ (n may vary), i.e. such that the projective representations of Q all come from linear representations of G. If one knows G, one may classify its linear representations by character theory. Of course one takes G minimal here. Then Schur calls G a representation group of Q (Darstellungsgruppe). As this term no longer sounds like what it is trying to convey, let us say instead that $\varphi\colon G \to Q$ is a Schur extension of Q. In general there is no unique Schur extension of Q, but Schur discovered that the kernel $M(Q)$ of φ is unique (up to canonical isomorphism). He baptized it the multiplier of Q (Multiplikator). Unfortunately he also called $H^2(Q, \mathbf{C}^*)$ the multiplier and identified it with $M(Q)$ by observing that the character group $\mathrm{Hom}(A, \mathbf{C}^*)$ of a finite abelian group A (such as $M(Q)$) is isomorphic with A.