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Notes on real and complex C^*-algebras, by. K. R. Goodearl, Shiva Mathematics Series, Shiva Publishing, Ltd., Cambridge, Mass., 1982, 211 pp., $28.95. ISBN 0-9068-1216-X

The author of this book states his purpose clearly: "W[e] have tried to present to the non-specialist a view into the subject by means of its most striking theorems." He does not even hint at the vast range of the subject but merely covers a few things well. In a similar vein this review is directed not at the expert but at those who want to know why so many mathematicians study (and write books about) C^*-algebras. The review will follow the book in requiring C^*-algebras to be unital (i.e. to have multiplicative identity elements). It will also assume complex scalars except to discuss the real case.

The story begins with two classes of concrete examples: one commutative and the other not. Let S be a compact Hausdorff space. Let $C(S)$ be the space of all continuous complex valued functions on S. Under pointwise addition and multiplication $C(S)$ is a commutative algebra. That is, it is a linear space which is also a commutative ring under the same additive structure and with the scalar and ring multiplications agreeing as one would expect. In addition $C(S)$ has two other elements of structure which turn out to be crucial to its study. It has a norm (the supremum or uniform norm) defined by

$$\|f\| = \sup\{|f(s)|: s \in S\} \quad \forall f \in C(S)$$

under which it is a complete normed algebra or Banach algebra. It also has an involution *: $C(S) \to C(S)$ defined by

$$f^*(s) = f(s)^- \quad \forall f \in C(S), \forall s \in S,$$

where the bar denotes complex conjugation. We will say more about involutions below, but for now we remark that an algebra with a (fixed) involution is called a $*$-algebra (pronounced star-algebra) and a Banach algebra with an involution is called a Banach $*$-algebra.