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ON THE YANG-MILLS-HIGGS EQUATIONS

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I. Introduction. The purpose of this note is to announce new results for the Yang-Mills-Higgs equations on R^3. These SU(2) Yang-Mills-Higgs equations are a set of partial differential equations where the unknown is a pair, $c = (A, \Phi)$, with A a connection on the vector bundle $E = R^3 \times su(2)$ and Φ a section of E. Here $su(2)$ =Lie alg. SU(2). These equations are

$$D_A * F_A + [\Phi, D_A\Phi] = 0, \qquad D_A * D_A\Phi = 0, \tag{1}$$

with boundary condition $\lim_{|x|\to\infty} |\Phi|(x) = 1$. Here, the notation follows [1]. That is, F_A is the curvature of A, D_A is the exterior covariant derivative on $\bigwedge T^* \otimes E$ and $[\cdot,\cdot]$ is the natural, graded bracket on $\bigwedge T^* \otimes E$: If ω, η are, respectively, E-valued p, q forms, then $[\omega, \eta] = \omega \wedge \eta - (-1)^{pq}\eta \wedge \omega$. The $*$ in (1) is the Hodge star on $\bigwedge T^*$ from the Euclidean metric on T^*. The norm $|\cdot|$ on $T^* \otimes E$;is that induced from the Euclidean metric on T^* and the Killing metric on $su(2)$.

Equation (1) is the variational equation of an action functional

$$\mathcal{A}(A, \Phi) = \frac{1}{2}\int_{R^3} \{|F_A|^2(x) + |D_A\Phi|^2(x)\}\, d^3x. \tag{2}$$

One is to consider $\mathcal{A}$ as a function on the set

$$\mathcal{C} = \{\text{smooth } (A, \Phi) : \mathcal{A}(A, \Phi) < \infty \text{ and } 1 - |\Phi|(x) \in L^6(R^3)\}. \tag{3}$$

$\mathcal{C}$ is topologized as follows [2]: Let θ denote the flat, product connection on E. The topology of $\mathcal{C}$ is defined to be the weakest for which the map sending $C = (A, \Phi) \in \mathcal{C}$ to

$$(A - \theta, \mathcal{A}(c)) \in \Gamma(T^* \otimes E) \times \Gamma(E) \times [0, \infty)$$

is continuous.

The topological group

$$\begin{aligned} \mathcal{G} &= \{\text{smooth, unitary automorphisms of } E\}, \\ &= C^\infty(R^3; \mathrm{SU}(2))/\{\pm 1\} \end{aligned} \tag{4}$$

acts continuously on $\mathcal{C}$ and leaves $\mathcal{A}$ and (1) invariant. The subgroup $\mathcal{G}_0 = \{g \in \mathcal{G} : g(0) = 1\}$ acts freely on $\mathcal{C}$. Let $\mathcal{B} = \mathcal{C}/\mathcal{G}_0$ denote the quotient. The functional $\mathcal{A}$ descends as a continuous, SO(3) invariant function on $\mathcal{B}$.

The relationship between $\mathcal{A}$ and $\mathcal{B}$ is described in the following theorems.

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