BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 10, Number 2, April 1984

LIMIT LINEAR SERIES, THE IRRATIONALITY OF M_g, AND OTHER APPLICATIONS

BY DAVID EISENBUD AND JOE HARRIS[1]

ABSTRACT. We describe degenerations and smoothings of linear series on some reducible algebraic curves. Applications include a proof that the moduli space of curves of genus g has general type for all $g \geq 24$, a proof that the monodromy action is transitive on the set of linear series of dimension r and degree d on a general curve of genus g when $\rho := g-(r+1)(g-d+r)=0$, a proof that there exist Weierstrass points with every semigroup of a certain class—in particular, on curves of genus g, all those semigroups with weight $w \leq g/2$ occur and a proof that the monodromy group acts as the full symmetric group on the g^3-g Weierstrass points of the general curve.

Curves will here be reduced, connected, and complex algebraic.

The study of general curves (Brill-Noether theory, etc.) and of moduli of curves depends on the degeneration of smooth curves to singular ones. Originally, the singular curves used were irreducible curves with nodes ([**G-H**] is a recent avatar) or, more recently, cusps [**E-H1**], but from the work of Mumford and others on the moduli space of stable curves it is apparent that reducible curves should be considered as well.

Unfortunately the degeneration of a linear series on a curve which degenerates to a reducible curve has not been well understood except in the particularly simple case of pencils; there the "limit" of the linear series, after removing base points, corresponds to an admissible covering, in the sense of Beauville, Knudsen and Harris-Mumford [**B, K, H-M**], of a curve of genus 0. The potential of a general theory is indicated, for example, by work of Gieseker [**G**].

In this announcement we describe the limits of linear series on some reducible curves and give some applications.

We call a curve *tree-like* if its irreducible components meet only two at a time, in ordinary nodes, in such a way that its dual graph (a vertex for each component, an edge for each intersection between distinct components) has no loops.

We say that a curve is of *compact type* if its (generalized) Jacobian is compact, or, equivalently, if it is tree-like and its irreducible components are all nonsingular.

Received by the editors November 8, 1983.

1980 *Mathematics Subject Classification*. Primary 14H10.

[1]The authors are grateful to the NSF, and the second author is grateful to the Alfred P. Sloan Foundation, for partial support of this work.