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THE HEAT EQUATION AND GEOMETRY OF CR MANIFOLDS¹

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It is well known that the trace of the heat semigroup for the Laplacian on a compact oriented Riemannian manifold has an asymptotic expansion whose terms are integrals of local geometric invariants; see [1, 3, 4] and their references. Entirely analogous results are true for the sublaplacian \square_b on a compact CR manifold. For simplicity, we state results here only for the case of a definite Levi form.

We suppose that the compact CR manifold M has definite Levi form and has been given a Hermitian metric and an orientation; thus there is an inner product in the space $\mathcal{E}^{p,q}$ of forms of type p, q . Let $\mathcal{H}^{p,q}$ be the completion and fix p . The operator

$$\bar{\partial}_b = \bar{\partial}_{b,q}: \mathcal{E}^{p,q} \rightarrow \mathcal{E}^{p,q+1}$$

has formal adjoint \mathcal{D}_b and gives rise to a nonnegative selfadjoint operator $\square_b = \square_{b,q}$ on $\mathcal{H}^{p,q}$ which extends the operator $\mathcal{D}_{b,q}\bar{\partial}_{b,q} + \bar{\partial}_{b,q-1}\mathcal{D}_{b,q-1}$. The operator $\square_{b,q}$ is hypoelliptic for $0 < q < n = \frac{1}{2}(\dim M - 1)$. In the special case that the metric is a Levi metric, there is a canonical metric connection due to Webster [9] and C. M. Stanton [5].

THEOREM 1. *For $t > 0$ and $0 < q < n$, the operator $\exp(-t\square_{b,q})$ has a smooth kernel $K_{t,q}$. On the diagonal, $K_{t,q}$ has an asymptotic expansion*

$$(1) \quad \text{tr } K_{t,q}(x, x) \sim t^{-n-1} \sum_{j=0}^{\infty} t^j K_{j,q}(x) dV(x), \quad t \rightarrow 0+,$$

where $\text{tr}: \text{Hom } \Lambda^{p,q} \rightarrow \Lambda^{2n+1}$ is the standard map and $dV(x)$ is the volume element. The functions $K_{j,q}$ are locally computable. If the metric is a Levi metric, then $K_{j,q}$ may be computed by evaluating a universal polynomial in the components of the curvature and torsion of the Webster-Stanton connection and their covariant derivatives calculated in normal coordinates.

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