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PROPER HOLOMORPHIC MAPPINGS

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Introduction. Let us recall that a mapping F: $X \to Y$ is *proper* if $f^{-1}(K)$ is a compact subset of X whenever $K \subset Y$ is compact. If X and Y are complex spaces, and if F: $X \to Y$ is a proper holomorphic mapping, then $F^{-1}(y_0)$ is a compact analytic subvariety of X for all points $y_0 \in Y$. Proper mappings between complex spaces were studied from the general point of view of complex spaces in the 1950s and early 60s (see Remmert-Stein [**78**]). Two results from this era are a factorization theorem of Stein [**88**] and the Remmert Proper Mapping theorem: *If f: $X \to Y$ is a proper mapping, and if $S \subset X$ is a subvariety of X, then $f(S)$ is a subvariety of Y.*

Here we consider a special case: proper mappings F: $\Omega \to D$ where $\Omega \subset\subset X = \mathbf{C}^n$ and $D \subset\subset Y = \mathbf{C}^N$ are smoothly bounded domains.[2] The letters Ω and D will always denote domains of $\mathbf{C}^n$, and a "proper mapping" will always be assumed to be holomorphic. (In many cases the same results are valid in the case where X and Y are Stein manifolds, although we will not emphasize this point.)

It is evident that a mapping F: $\Omega \to D$ is proper if and only if f maps $\partial\Omega$ to ∂D in the following sense:

$$\text{if } \{z_j\} \subset \Omega \text{ is a sequence with } \lim_{j\to\infty} \operatorname{dist}(z_j, \partial\Omega) = 0, \text{ then } \lim_{j\to\infty} \operatorname{dist}(f(z_j), \partial D) = 0.$$

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[2] In this case proper mappings are also known as "finite mappings".