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SURGERY AND BORDISM INVARIANTS

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Introduction. The approach used here to relate the two subjects in the title is best explained in terms of three "machines".

Machine (1) is the "L-theory machine", or "surgery machine"; on being fed a discrete group G and homomorphism $w\colon G \to Z_2$, it produces a spectrum $\mathbb{L}_{\cdot}(G,w)$ whose homotopy groups are the surgery obstruction groups (choose your favourite version),

$$\pi_n(\mathbb{L}_{\cdot}(G,w)) = L_n(G,w) \quad \text{for } n \in \mathbf{Z}.$$

Machine (2) is the "bordism theory machine": on being fed a CW-space B and vector bundle γ on B, it produces a bordism spectrum (or Thom spectrum) $M(B,\gamma)$. The homotopy groups $\pi_n(M(B,\gamma))$ are the bordism groups of closed smooth manifolds N^n equipped with a bundle map from the normal bundle ν_N to γ.

This note will describe a third machine, obtained by welding together the previous two. (The aim is to extend the theory of the "generalized Kervaire invariant": cf. [**1**, **2**].)

Description of Machine (3).

Input. The following input data are required:

— a group G and homomorphism $w\colon G \to Z_2$, as for Machine (1);

— a CW-space B and bundle γ on B, as for Machine (2);

— a principal G-bundle α on B and an identification j of the two double covers of B arising from these data. (They are the orientation cover associated with γ, and the double cover induced from α via w.)

Output. Machine (3) produces a spectrum $\mathbb{L}^{\cdot}(G,w;B,\gamma;\alpha,j)$ (informally: $\mathbb{L}^{\cdot}(B,\gamma)$) and maps of spectra

$$\mathbb{L}_{\cdot}(G,w) \to \mathbb{L}^{\cdot}(B,\gamma) \leftarrow M(B,\gamma).$$

Like Machines (1) and (2), Machine (3) is functorial: Given two input strings $(G,w;B,\gamma;\alpha,j)$ and $(G',w';B',\gamma';\alpha',j')$, and

— a map $f\colon B \to B'$ covered by a bundle map $\gamma \to \gamma'$;

— a homomorphism $h\colon G \to G'$ so that $w' \cdot h = w$;

— an identification of principal G'-bundles on B,

$$h_*(\alpha) \cong f^*(\alpha'),$$

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