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Q VALUED FUNCTIONS MINIMIZING DIRICHLET'S INTEGRAL AND THE REGULARITY OF AREA MINIMIZING RECTIFIABLE CURRENTS UP TO CODIMENSION TWO

F. J. ALMGREN, JR.[1]

We announce several results of an extensive study [**A**] of the size of singular sets in oriented m dimensional surfaces which are area minimizing in $m+l$ dimensional Riemannian manifolds. Our principal result is that the Hausdorff dimension of such a singular set does not exceed $m-2$. Examples show this is the best possible such general estimate when $l \geq 2$, i.e., when branching singularities are possible. The general existence of such surfaces of least area is well known in a variety of settings [**F**, 5.1.6].

In order to obtain estimates on branching of area minimizing surfaces we were led to use Taylor's expansion in terms of first derivatives at 0 to approximate the nonparametric area integrand by Dirichlet's integrand. Accordingly, we study branched coverings of regions in $\mathbf{R}^m$ which are graphs of multiple valued functions minimizing the integral of Dirichlet's integrand. As a central estimate in our analysis of area minimizing surfaces we show that the Hausdorff dimension of the branch set of such a minimizing covering does not exceed $m-2$.

To state several results in more detail we use the terminology of [**F**]. Suppose that A is a bounded open subset of $\mathbf{R}^m$ with smooth boundary, and let k, l, m, n, Q be positive integers with $k \geq 3$, $l \leq n$, and $m \geq 2$.

INTERIOR REGULARITY OF ORIENTED AREA MINIMIZING SURFACES. *Suppose N is an $m+l$ dimensional submanifold of $\mathbf{R}^{m+n}$ of class $k+2$ and that T is an m dimensional rectifiable current in $\mathbf{R}^{m+n}$ which is absolutely area minimizing with respect to N. Then there is an open subset U of $\mathbf{R}^{m+n}$ such that $\operatorname{spt} T \cap U$ is an m dimensional minimal submanifold of N of class k and the Hausdorff dimension of $\operatorname{spt} T \sim (U \cup \operatorname{spt} \partial T)$ does not exceed $m-2$.*

For such area minimizing T we have additionally

SINGULARITY STRATIFICATION BY TANGENT CONE TYPE. *Whenever $p \in \operatorname{spt} T \sim \operatorname{spt} \partial T$ and S is an oriented tangent cone to T at p then*

$$P(S) = \mathbf{R}^{m+n} \cap \{x : \theta^m(\|S\|, x) = \theta^m(\|S\|, 0) = \theta^m(\|T\|, p)\}$$

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